JEHAN DE MEUR'S MUSICAL THEORY AND THE MATHEMATICS OF THE FOURTEENTH CENTURY

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In the world of music, theory generally follows practice; but in Jehan de Meur's treatise on the notation of musical rhythm we find a most unusual case of a theory (informed by a new approach in mathematics) which opened up hitherto unheard-of possibilities in musical practice not to be realized until later. He consolidated his intricate, yet entirely imaginary rhythmical patterns in the second book of his *Notitia artis musicae* entitled *Musica Practica*, dated 1321. Jehan's treatise is well-known among historians of musical theory, but its close connections to the contemporanous Mertonian Calculatory tradition have not previously been noticed. I propose to explain this remarkable reversal of the customary course of events, with musical works nourishing theories rather than the other way round, by examining Jehan's musical thought in the context of the Mertonian procedures in quantifying qualities. These novel procedures were stimulated by the new fourteenth-century atmosphere of unleashing human creative power and gave free reign to imagined possibilities, in the name of God's *potentia absoluta*.

The Mertonian tradition

Historians of science have repeatedly pointed out the unitary character of late medieval learning and have noted in particular the wide diffusion of the Merton Calculatory tradition within other disciplines ranging from logic, natural philosophy, theology and medicine.[1] However, the field of musical theory is conspicu-

[1] See especially John Murdoch, "From Social into Intellectual Factors: An Aspect of the Unitary Character of Late Medieval Learning," in J. Murdoch and E. Sylla (ed.), *The cultural Context of Medieval Learning* (Dordrecht and Boston: Reidel, 1975), pp. 271-339. See also: Edith Sylla, "The Oxford Calculators in Context," *Science in Context* 1, 1987, p. 257.

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ously absent in these accounts. In my doctoral dissertation *Music in the Age of Ockham: The Interrelations between Music, Mathematics, and Philosophy in the Fourteenth Century*[2] I have examined a range of unexpected connections between science and music during that creative period, of which I wish to pursue a particularly striking one in the present paper. By scrutinizing the affinity between the musical and the mathematical discourses of the period in question, we shall demonstrate the presence of *bona fide* Mertonian metaphysical presuppositions, terminology and operative principles in a treatise about musical rhythm, that is, motion in time, that seems to precede any known mathematical treatise of the Mertonian school. Surprisingly perhaps, it was written at the Sorbonne by a French music theorist who was also a well known mathematician and astronomer. Jehan's activity as an astronomer, especially his original contributions to the diffusion of the Alfonsine tables in his astronomical writings from 1321 onwards, attests to his experience with concrete quantification and measurement of time and motions, in a discipline that was, on the one hand, part and parcel of the academic *Quadrivium*, and, on the other, associated with such practical needs as rendering accurate a perpetual calender as well as devising means for telling the time at different seasons of the year.[3]

Jehan applies numbers and fractions to physical quantities in his short treatise on mechanics, *De moventibus et motis*, encapsulated within the fourth book of his famous mathematical work of 1343, the *Quadripartitum numerorum*.[4] Here Jehan devotes himself solely to abstract comparisons of different rectilinear and circular motions, sometimes commensurable, sometimes incommensurable. But all the motions concerned are simple or uniform. No reference is made to natural motions involving acceleration or deceleration, or motion against a resisting force. In fact, there is no single reference throughout the whole treatise to these two key concepts of Mertonian mathematical physics, namely, acceleration and resistance. G. L'Huillier, in her new edition of Jehan's *Quadripartitum*, stresses the conservative tone of Jehan's mechanics. Jehan, she argues, clearly adheres to the simple Aristotelian theory and omits all mention of the complicated yet widely accepted logarithmic formula proposed by the

[2] D. Tanay, *Music in the Age of Ockham: The Interrelations between Music, Mathematics, and Philosophy in the Fourteenth Century* (Doctoral Dissertation, University of California, Berkeley, 1989).

[3] E. Poulle emphasizes not only Jehan's knowledge of trigonometry and geometry, but also his deep concern with adjusting theoretical calculations to practical usage and with recording astronomical observations, both of which were, in general, very rarely practiced during the Middle Ages. See E. Poulle, "John de Murs," in *Dictionary of Scientific Biography*, vol. 7 (New York, 1973), pp. 128-133.

[4] For a new edition and study of this source, see G. L'Huillier, *Le Quadripartitum Numerorum de Jean de Murs* (Mémoires et Documents Publiés par la Société de L'École des Chartres, vol. 32, Geneve/Paris, 1990).

Mertonian Calculator Thomas Bradwardine in his *Tractatus de proportionibus* of 1328.[5] According to L'Huillier, a good number of French mathematicians, such as J. Buridan, were reluctant at the time to adopt Bradwardine's logarithmic formula. Although L'Huiller does not explain her finding, it seems that French mathematicians rejected Bradwardine's exponential relations as a legitimate interpretation of the Aristotelian relation between velocity and forces,[6] perhaps because it jarred with Aristotle's explicit warning against the application of overtly complicated mathematics to physics (such as the compound proportions that Bradwardine applied in his law of dynamics).[7] Reaffirming the simple Aristotelian relation, Jehan de Meur sided with Buridan, so argues L'Huiller, and maintains that Jehan de Meur cannot be counted among pioneers of scientific research in the fourteenth century, at least with regard to physics proper.[8] Viewed against this background, the very presence of Calculatory methods and notions in the musical discourse of Jehan de Meur (including among others, the compound notion of proportion of proportion) is rendered intriguing. It raises several questions concerning the early history of the Mertonian school, such as its possible sources, its cultural resonance and social impact, the position of Jehan de Meur in the history of mathematics and mathematical physics, and the relation between Jehan's mathematical, astronomical and musical works. From the perspective of music history the ensuing discussion will demonstrate that the new mathematics of the fourteenth century played a significant role in the gradual emergence of a new concept of musical esthetics that paved the way to the acceptance of musical imperfection and disharmony as constitutive parts of the new notion of beauty in music. What this

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[5] For Jehan's analysis of motion see L'Huillier (n. 4), *Le Quadripartitum*, pp. 485 and 508-511. For Bradwardine's law of dynamics, see L. Crosby, *Thomas of Bradwardine: His Tractatus de Proportionibus: Its Significance for the Development of Mathematical Physics* (Madison, Wisconsin: Wisconsin University Press, 1961), pp. 75-86.

[6] According to Aristotle, velocity was in direct proportion to the moving force and inverse proportion to the resistence. This can be expressed (with anachronistic notation) as $V = F/R$ (V = velocity, F = force, R = resistance). Bradwardine corrected an immanent difficulty of Aristotle's rule, namely, that motion is actualized even when $R > F$, by proposing his alternative rule that involves the rather intricate notion of compound proportion:

$$F_2/R_2 = (F_1/R_1)^{V_2/V_1}$$

[7] At most, according to Aristotle, mathematical figures were useful for physics, in order to represent a body at rest or in periodic motion, that is, regular, simple, and uniform celestial motions. Furthermore, representation of change in intensity or acceleration was impossible, for the very notion of a rate of change was for him impossible: "There cannot be motion of motion, or becoming of becoming, or in general, change of change." (Aristotle, *Physics*, Book 5, Chapter 2, 225b15).

[8] L'Huillier (n. 4), *Le Quadripartitum*, p. 29. Yet, as far as pure mathematics is concerned, L'Huillier argues that Jehan was not only familiar with concurrent developments but actually inspired them and, to a certain extent, even anticipated them. See *ibid.*, pp. 23-29.

means is that the revolt, embodied in the Mertonian quantification of qualities, against firm Aristotelian prohibitions has quite unexpected consequences in a domain seemingly far removed from mathematics. The duration of musical notes, hitherto considered in purely qualitative terms, became subject to quantification in the same way that latitudes and motions were at the time.

Assessing influences

Before exploring the possible philosophical and mathematical content of Jehan's theory of rhythmic notation and before attempting to scrutinize the possible contribution of fourteenth-century mathematics to the evolution of a new ideal of music, let me first delineate the boundaries of my arguments.

No direct evidence has yet been found to warrant the existence of personal links or even indirect contact between Jehan and the Merton Calculators. This might be a consequence of the paucity of available biographical data on the individuals concerned. My argument, therefore, rests solely on textual analysis. Most pressing, for the sake of the ensuing arguments, is the question of determining the direction of dependency: was it Jehan's advanced musical thought that was nourished by Mertonian mathematical knowledge, or conversely should we attribute the mathematical breakthrough of the Mertonians to contemporaneous musical erudition? Perhaps a third alternative exists, namely, that both Jehan's musical innovations and the Calculators' new mathematics emerged independently as two distinct manifestations of a common source? To set the scene for probing these questions further, we need first to eliminate the possibility that the similarity between the Calculators' works and Jehan's musical thought was merely accidental, that is, no more than an insignificant coincidence of two separate and unrelated developments.

Historians of science have argued time and again that the interest in quantification of qualities was by no means an idiosyncratic interest of the Mertonian Calculators. It was induced, if not actually initiated, by theological discussions of the infusion of *caritas*, and/or by medical-pharmaceutical quests for enhancing medical treatments by compounding drug effects, or by studies of the intensification and propagation of light found both in optics and outside optics in debates related to divine illumination. That is to say that these attempts to rationalize and account for processes such as motion, growth, intensification or remission attest to a widespread, central and general concern of their time. Musical and mathematical theorizations of variability must then be seen as part of this broader context of fourteenth-century preoccupation with quantification and measurements. It would be misleading, therefore, to interpret the resemblance between the musical and mathematical discourse as a mere coincidence.

Indirect circumstantial evidence may support the conjecture that the two activities stemmed from a common source. M. Clagett notes the influence of Gerard of Brussels' *Liber de motu* on the kinematical works of Thomas Bradwardine, the founder of the Mertonian Calculatory tradition.[9] Gerard, who wrote his treatise on motion between 1187 and 1260 (probably at the the university of Paris), endeavored to reduce variability to uniformity – or disorder to order – by converting the varying curvilinear velocities of the points of geometrical figures in rotation into a simple uniform rectilinear motion. According to M. Clagett, Gerard's famous proposition: "Any part as large as you wish of a radius describing a circle ... is moved equally as its middle point. Hence the radius moves equally as its middle point" probably anticipated William Heytesbury's mean speed theorem, which likewise translated a motion composed of infinitely varying velocities into a uniform motion. Since Jehan worked at the Sorbonne where he was already a master of arts in 1321, we could assume that he was familiar with thirteenth-century French kinematical works. Indeed, his earliest mathematical work, the *Canones tabule tabularum* of 1321, mentions a squaring of the circle, quoted also by Gerard in his *Liber de motu* (although it is based on the earlier translation of Gerard of Cremona). Jehan could just as well have used this source, or Archimedes' *De mensura circuli* as translated by William of Moerbeke, to which he refers in his *De arte mensurandi* of 1344. The chance that Jehan's idea was somehow motivated directly by the Mertonian mathematical revolution is further weakened by the fact that he ignored the innovations of the Calculators in his essay on mechanics (in book IV of his *Quadripartitum numerorum* of 1343).

Finally and conversely, no evidence can be adduced to bear on the possibility that the Calculators absorbed in their own works Jehan's notions and principles. There is, notwithstanding, a significant reference to a theory of music in Bradwardine's *Tractatus de proportionibus* of 1328. Bradwardine refers to the ancient Pythagorean theory of harmony as authoritative corroborating evidence for measuring continuous quantities (in this case musical strings) by discrete numbers.[10] Musical harmonies were expressed by simple ratios, such as 2:1 or 3:2 that describe the proportional relation between the length of two different strings. Bradwardine here admits that the philosophical roots of his mathematical approach to physics stem from the Platonic Pythagorean tradition, a tradition that was subdued in the realm of natural philosophy during the age of Scholasticism, yet prevailed in the domain of music side by side with Scholastic musical thought throughout the Middle Ages. Bradwardine's notion of music, therefore,

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[9] M. Clagett, "Gerard of Brussels," in *Dictionary of Scientific Biography*, vol. 10 (New York, 1973), p. 360.

[10] Crosby (n. 5), *Thomas of Bradwardine*, pp. 74-75.

seems to have been tethered more to Classical textbook expositions of the science of music, rather than to the surrounding soundscape or contemporaneous developments in musical theory.

Let us now turn from the question of "influence" to the deeper and more challenging question of grounding Jehan de Meur's apparently autonomous theory of rhythmic notation in the context of the new fourteenth-century theories of dynamics and kinematics. By way of an introduction let us scrutinize the tacit import of the "avant garde" ideas of Jehan's *ars notandi* or *ars nova*. In what ways are those ideas categorically different when compared with those of his predecessors?

The Franconian notation of rhythmical patterns

Since the thirteenth century, musical time was commonly divided into uniform cycles or periods. Each such cycle was conceived of as a perfect rhythmic whole (*totus perfectus*). In practice, this basic whole, the regulative principle of measuring time in music, was articulated and variegated through substituting the whole with its parts. In the thirteenth century the partition of the rhythmic whole was confined to either three equal parts or two unequal parts: one longer, the other shorter, in the proportion of 2:1. The three equal parts that resulted from this division could be further subdivided into either three equal parts or two unequal parts again, in the proportion of 2:1. In light of the Scholastic-Thomistic propensity for symbolic presentations, music theorists conceived the rhythmic perfect whole as an hypostatization of divine perfection: it is a Trinity that is a unity. Franco of Cologne, the leading theorist of the thirteenth-century *ars antiqua* summarizes and conceptualizes in his *Ars cantus mensurabilis* ca. 1260 the principles of notating rhythmic values.[11] Fig. 1 represents the basic vocabulary of the thirteenth-century Franconian system of notation as expounded in Franco's treatise. The system consists of three species of note-values: *longa*, *brevis* and *semibrevis*. Oddly enough, the system is both economical and cumbersome. Two graphic signs account for the distinction between a long and a short value, while short values are further differentiated into breve and semibreve. But

[11] The authority of Franco's theory was recognized already in his own days. Setting forth the revolutionary standard of musical notation, namely, that different rhythmic durations of individual musical notes should be notated by distinct graphic symbols, Franco lay the foundation of Western musical notation. His theory was widely distributed and conceived of as highly authoritative. His system of notation, in spite of its obvious deficiencies, served as the base for rhythmic notation until the seventeenth century. For a detailed biography and bibliography, see A. Hughes' article: "Franco of Cologne," in Stanley Sadie (ed.), *The New Grove Dictionary of Music and Musicians*, vol. 6 (London, 1980), pp. 794-797.

each of these three signs actually denotes more than just one rhythmic value: the figure of the long could signify a prolongation throughout three units of time (i.e., three breves) or throughout two units (i.e., two breves).

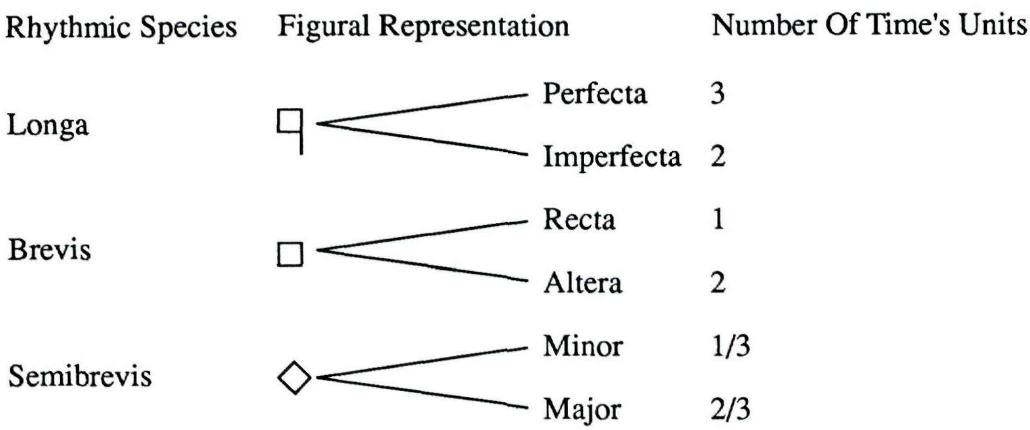

Figure 1

Likewise, the figure of the breve could indicate two different short values, one containing a single part, the other containing two such parts or units. Franconian notation is beset by the shortcomings of its immanent ambiguities. One could attribute such inadequacies to the immaturity of rhythmic notation. Yet it might be more revealing to situate Franconian semiotic behavior in the conceptual framework of Aristotelian tenets. Theoreticians who worked within the frame of the Aristotelian Scholastic mode of thought abstracted the notion of duration into the general distinction between the long versus the short. Under the spell of Aristotle's qualitative physics, long and short were taken as qualities rather than quantities. Theorists of the thirteenth century conceptualized durations as "essential predicates" of a "subject" which is sound itself. Once reasoned into the status of predicates and abstracted into the only two options of duration, the long and the breve become categorically differentiated as distinct species. As shown in Fig. 1, each species has its own sub-classes, but the long and the breve do not belong to a larger whole in the same species, which contain both, and which would have made possible a quantitative comparison between them. In Franco's *Ars cantus mensurabilis*, this qualitative approach is expressed as follows: "Of simple figures there are three species: long, breve and semibreve, the first of which has three varieties: perfect, imperfect and duplex ... The breve may be divided into *brevis recta* and *brevis altera*. Regarding the semibreve, one is said to be major, the other minor."[12]

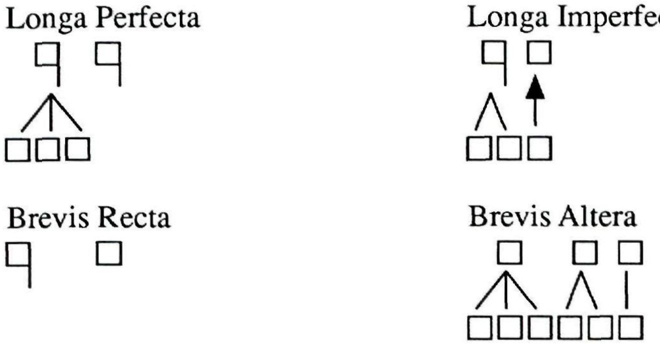

Figure 2

The valuation of the Franconian figures is determined by their surrounding context. As indicated in Fig. 2, a long before a long is a perfect long (consisting of three units or parts) while a long before a breve is an imperfect long consisting of two units or parts. In this case the breve that follows the long causes its imperfection. The Aristotelian notion of the contrariety between being and privation infiltrates the discourse of thirteenth-century music theorists, in their distinction between the two types of longs. The binary imperfect long is one deprived of perfection; imperfection befalls when one part of the long is extracted and figured independently, whereby the long becomes binary rather than ternary and therefore loses its perfection. An imperfect long has no separate reality (Fig. 2): it never appears alone; as a rule it is always followed by a breve to round off the cycle of perfection. Turning from the long to the breve, we see in Fig. 2 that a breve before or after a long consists of one unit of time, but if followed by another breve the second breve is altered and equal to two units of time.[13] Thus, an altered breve is quantitatively equal to an imperfect long, but the mathematical aspect is not relevant, because imperfect long and altered breve are qualitatively distinct. The breve can be replaced by three equal

[12] G. Reaney and A. Giles (ed.), *Corpus Scriptorum de Musica*, vol. 18: Franco de Colonia, *Ars cantus mensurabilis* (Rome, 1974), p. 29: "Simplicium tres sunt species, scilicet longa, brevis et semibrevis. Quarum prima in tres dividitur: in longam perfectam, imperfectam et duplicem longam. Brevis autem licet in rectam et alteram brevem dividatur. Semibrevis autem alia maior, alia minor dicitur."

[13] For a complete exposition of the Franconian system, see Franco of Cologne, "Ars cantus mensurabilis," in O. Strunk (ed.), *Source Reading in Music History* (London: Faber & Faber, 1952), pp. 139-159.

parts or two unequal parts (called *semibrevis*), but it cannot be imperfected by its third part. That is to say that at this point in music history the rhythmic hierarchy is based on three levels of duration: the long, the breve and the semibreve. But these levels operate independently: they do not relate mathematically as parts to a larger whole, and each level is governed by its own set of rules. Such semiotic behavior fits well with the scholastic understanding of rhythmic values as distinct qualities, rather than quantities.

Qualities quantified

Historically, the turning point at which Jehan de Meur is situated still carries a conceptual predisposition towards the Aristotelian qualitative discourse of the long and breve. However, at the same time, the shift towards the quantitative discourse is already in view. It highlights, then, a general tendency in fourteenth-century philosophy and mathematics to depart from the Aristotelian ban on mixing categories.[14]

The inherent import of this departure became highly significant when the Merton Calculators pioneered their imaginary quantification of qualities. In their terminology, quantification is coined as the dynamics of intention and remission of forms (*Intensio et remissio formarum*). The Mertonian Calculator John Dumbleton conceptualized and mathematized the notion of a latitude of different degrees of intensities. He identified the latitude of a given quality with the given concrete degree of intensity (called *gradus*) and developed a one-dimensional coordinate system to represent the latitude of form. The increase of intensity was seen as analogous to a geometric line that was limited at one end, by the *minima* or zero degree of intensity and at the other end by the *maxima*. The various parts of the line, however minute, represent degrees of intensity. Intensification in this context means passing through all the degrees of intensities between the given and the gained.[15] Thus the Mertonians echo the ontology of Duns Scotus, which allows quality to be contracted in different degrees of intensities and which calls for a mathematical analysis of intensification or remission by addition or subtraction of parts of the quality in question.[16]

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[14] For the historical background of the dissolution of the Aristotelian world view and the resulting early experimentations with mixing genera or qualities, see E. Sylla, "Medieval quantification of qualities: The Merton School," *Archive for the History of Exact Sciences* 8, 1971, pp. 9-39.

[15] E. Sylla, "Medieval Concepts of the Latitude of Forms: The Oxford Calculators," *Archives d'histoire doctrinale et littéraire du Moyen Age* 40, 1973, pp. 251-269.

[16] A. Funkenstein, *Theology and the Scientific Imagination from the Middle Ages to the Seventeenth Century* (Princeton: Princeton University Press, 1986), pp. 307-309.

Both Scotus and the Mertonians eroded the Aristotelian maxim of forms as being immutable and indivisible and therefore unquantifiable. In so doing, they not only committed the vice of *metabasis* so strongly prohibited by Aristotle, but also undermined the latter's fundamental condition of reasonable discourse. Strikingly enough, it is Jehan's musical theory that may affirm an explicit link with the new anti-Aristotelian mode of thinking. In other words, what we find in Jehan's *Notitia* is an early presentation of the Mertonian theory of intensification or acceleration, applied to the musical being, conceived of as motion measured against the continuum of time.[17] While for his predecessors long and short were categorically incompatible, Jehan's theory is grounded upon the precept of the continuity and commensurability of rhythmic values. Jehan says: "As for times, one is greater, the other is lesser. The greater time, which has a longer motion, and the lesser time, which has a shorter one, are measured (other things being equal) according to one and the same dimension. For they are not different in species, since the greater and the lesser do not alter species."[18]

"More" or "less" of the same quality, then, does not change the species; since both belong to a larger whole of the same species. In the Mertonian theory this whole, to recall, was the latitude of the quality, abstracted and notated by a geometrical line. A homological model is evident in Jehan's *Notitia*, where the rhythmic whole is represented by the maximal note-value, called in Jehan's text the *longissima*. Neither the term nor the concept have a previous history and have never been witnessed in concrete music notation before. Jehan demonstrates the latitude and the limits (*termini*) of a musical sound by the following table (Fig. 3), where the latitude of a prolonged sound is divided into four grades of perfection, and each grade is further subdivided into three parts in the relation of 3:2:1. The extreme low end of the continuum is now called, in music, "minima."

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[17] Ulrich Michels, *Corpus Scriptorum de Musica*, vol. 17: Johannes de Muris, *Notitia artis musicae. Compendium musicae practicae, with the treatise of Petrus de Sancto Dionysio* (Rome, 1972), (hereafter Johannes de Muris), p. 65: "Ut in primo <libro> ostensum est, vox generatur cum motu, cum sit de genere successivorum. Ideo quando fit, est, sed cum facta est, non est ... Igitur vocem necessario oportet tempore mensurari. Est autem tempus mensura motus. Sed hic tempus est mensura vocis prolatae cum motu continuo."

[18] *Ibid.*, p. 66: "Temporis aliud maius aliud minus: maius, quod motum prolixiorem, minus, quod breviorem habet ceteris eisdem, secundum unam dimensionem metitur. Haec autem specie non differunt, nam maius et minus speciem non variant." Michels suggests that Jehan speaks in this passage about the general notion of time (as distinct from musical time): "auf den Zeitbegriff im allgemeinen, nicht im musikalisch speziellen Sinn bezieht sich die Erklärung zum tempus maius und minus bei Muris." (U. Michels, "Die Musiktraktate des Johannes de Muris," *Beihefte zum Archiv für Musikwissenschaft* 8, 1970, p. 3.) I interpret the passage as referring to musical time. Therefore, the term *specie* in the above quotation refers to "short" and "long" prolongations ("maius, quod motum prolixiorem, minus, quod breviorem habet ceteris eisdem, ...) and not, as implied in Michels's commentary, to "perfect" and "imperfect" rhythmical values.

	3	81	longissima		
	2	54	longior		primus gradus
	1	27	longa	idem	
	3	27	perfecta		
	2	18	imperfecta		secundus gradus
	1	9	brevis	idem	
	3	9	brevis		
	2	6	brevior		tertius gradus
	1	3	brevissima	idem	
	3	3	parva		
	2	2	minor		quartus gradus
	1	1	minima		

Figure 3

The term itself comes from Aristotle who explains in his *De Caelo* that not only is there a minimum time and a maximum speed for the motions of the heavens, but also that for every action (as walking or playing the lyre), there is a minimum time and a maximum speed.[19] Thus Aristotle provided the very weapon with which his system was undermined, allowing the idea of a minimum quantitative dimension for the existence of natural substances, side by side with his notion of the absolute continuity of matter. As terms belonging to the vocabulary of the Calculators' conceptual language of intension and remission of forms, minima and maxima were used abstractly, that is, *secundum imaginationem*, with a logical intent only. They therefore could denote acceleration or intensification to infinity, and diminution to zero quantity.[20] Jehan, however, was dealing with a concrete natural phenomenon – a musical sound. Conceived as a *forma naturalis*, argues Jehan, a musical sound, measured by time, has its specific latitude, limited by a physical minima and a maxima. A physical minima, unlike a mathematical minima or a point, is indivisible in spite of having a positive length, perhaps a very short one. Consequently, according to Jehan, a sound, as all other natures, demonstrates that in nature (as opposed to mathematics) there are neither infinitely big nor infinitely small magnitudes:

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[19] Aristotle, *De caelo*, Book 2, Chapter 6, 288b30-289a4.

[20] The Calculators were not unanimous as to the understanding of the procedure involved in quantifying intension and remissions. For a panoramic view of their different concepts of measurement see M. Clagett, "Richard Swineshead and Late Medieval Physics," *Osiris* 9, 1950, pp. 131-161.

> Since the voice or musical sound measured by time constitutes a union of two forms: the natural and the mathematical. Even though according to the one (the mathematical) the division is endless, according to the other (the natural) its division must end somewhere. Just as for all things by nature there is a limit in magnitude and augmentation, so also in smallness and diminution. Natural things prove that nature is limited by a maximum and minimum. The voice, being in itself a natural form to which quantity is joined by accident, must have limits for its division, the latitude of which cannot be surpassed by any sound, however fleeting. We wish to understand these very limits by reason.[21]

Jehan refers clearly to the Aristotelian notion of *minima naturalia* and turns it into an avenue other than atomism, to concrete quantification. Furthermore, In Jehan's quantified notion of rhythmic duration all the longer degrees are measured by the rhythmic *minima*, and any degree representing a note-value is the sum of its constituent parts.[22]

Jehan's table (Fig. 3) implies not only the idea of commensurability of various lengths of time but also previews a multitude of possible rhythmic combinations. Such combinations would make the texture of parts and whole more subtle. For example, Jehan loosened the restricted and rigid rhythmic combinations of his predecessors. He legitimized the procedure of imperfection not only within each of the four grades, but also by crossing the boundaries between the grades. The rhythmic long, for example, could now be imperfected or diminished not only by its immediate part – the breve – but also by a remote part, namely the semibreve. Interestingly enough, in Jehan's time, some of those rhythmic varieties did not yet have a musical counterpart. This is an important issue, which will be further elaborated upon later.

The wealth of variability inside the latitude which is evident as a possibility in Jehan's notion of musical time also permeates the Mertonian discourse. The affinity between Jehan's refinements of musical time and the Calculators'

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[21] Johannes de Muris, *Notitia* (n. 17), p. 69: "Quoniam ergo vox tempore mensurata unionem duarum formarum, naturalis scilicet et mathematicae, comprehendit, licet quod ratione alterius fractio non cessaret, tamen propter aliam vocis divisionem necessarium est alicubi terminari. Nam sicut omnium natura constantium positus est terminus et ratio magnitudinis et augmenti sic parvitatis et diminuti. Demonstrant enim naturales, quod natura ad maximum et minimum terminatur. Vox autem est per se forma naturalis iuncta per accidens quantitati. Igitur oportet eam habere terminos fractionis, quorum latitudinem nulla vox quantacumque frangibilis valeat praeterire. Hos autem terminos volumus comprehendere ratione."

[22] Like Dumbleton, Jehan gave distinct reality to the latitude of given qualities and identified the latitude with the degree of intensity in a given instantiation. For Jehan the quality in question was the duration of note value. Furthermore, Jehan seems to be sensible to the distinction between physics and mathematics. In his view, the fact that a sound can be measured by mathematics does not entail any identical structure between them. While in the abstract, mathematical measure can be applied to a physical being, rhythmical divisions are empirically limited by the nature of sound and voice themselves.

analyses of motion is founded on the shared attempts to introduce order into the disturbing problem of variability and disorder. Both mapped out all the possible types of different distributions of qualities throughout given segments of space or time. The Calculators emphasized the very distinction between uniform (*uniformis*) and difform (*difformis*) distribution; namely, they distinguished between subjects manifesting a quality in uniform intensity or a movement in uniform velocity (distribution of a quality in time) and subjects manifesting several intensities of a quality. The class of difform distributions was further divided into different types of non-regular distributions. In discussing velocity the most common distinctions were between *motus uniformis* (uniform motion), *motus difformis* (difform motion), *motus uniformiter difformis* (motion in which the velocity increases uniformly), and *motus uniformiter difformiter difformis* (motion in which the acceleration increases uniformly).[23] These efforts towards rationalization by means of verbal arguments have clear correspondence in Jehan de Meur's *Notitia artis musicae*. Like the Calculators, Jehan sorts out types of distributions throughout a definite extension in the dimension of time (the *longa*), axed upon the field of rhythmic variability. His classifications are based on the distinction between perfect division (division into three equal parts) and imperfect division (division into two equal parts). In sorting out internal differences in the composition of the rhythmic long (Fig. 4), Jehan distinguished between: 1) *longa perfecte perfecta* – a perfect whole consisting of three perfect breves which contain nine semibreves, 2) *longa perfecte imperfecta* – one whole consisting of two perfect breves, altogether six semibreves, 3) *longa imperfecte perfecta* – a long diminished or imperfected by one or two of its nine constitutive semibreves, and 4) *longa imperfecte imperfecta*, which is diminished or imperfected by one breve and one semibreve. The terminological resemblance between the distinctions in mathematics and music is apparent, as is the resemblance of content.[24]

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[23] M. Clagett, *Science of Mechanics in the Middle Ages* (Madison, Wisconsin: Wisconsin University Press, 1959), pp. 247-248: "Motuum localium quidam est uniformis, quidam difformis ... Motus uniformis quo ad tempus est motus alicuius mobilis quo ipsum mobile in omni parte equalitemporis pro quo illud movetur pertransit spatium equale. Motus difformis quo ad tempus est motus mobilis quo ipsum mobile in una parte temporis plus pertransit quam in alia parte temporis sibiequali. Motus difformis quo ad tempus est duplex; non quidam est uniformiter difformis, quidam difformiter difformis. Motus localis uniformiter difformis describitur a calculatoribus sic: Motus localis uniformiter difformis est motus difformis, cuius quacunque parte signata medius gradus illius partis per equalem latitudinem excedit extremum remissius eiusdem sicut ipse ab extremo intentiori illius partis exceditur. Motus difformiter difformis est motus difformis non existens uniformiter difformis."

[24] It is possible that Jehan and/or the Calculators adapted Boethius' division of numbers into *pariter par*, *pariter impar*, *impariter par*, *impariter impar*. The point is that in the early fourteenth century such traditional schemes of classification were applied to the heretofore inconceivable realm of variability and contingency.

Rhythmical patterns beyond tradition

Let us take a closer look into *longa imperfecte perfecta* and *longa imperfecte imperfecta*. Both, once again, represent a new option of diminution or imperfection, namely imperfection not only by immediate and simple parts, but also by the parts of parts (Fig. 4). In other words, Jehan's rhythmic ideas imply the operation not only with simple proportions (the proportion between a long and a breve – the immediate part of the long), but also with compound proportion, or proportions of proportions (the proportion between a long and a semibreve).[25]

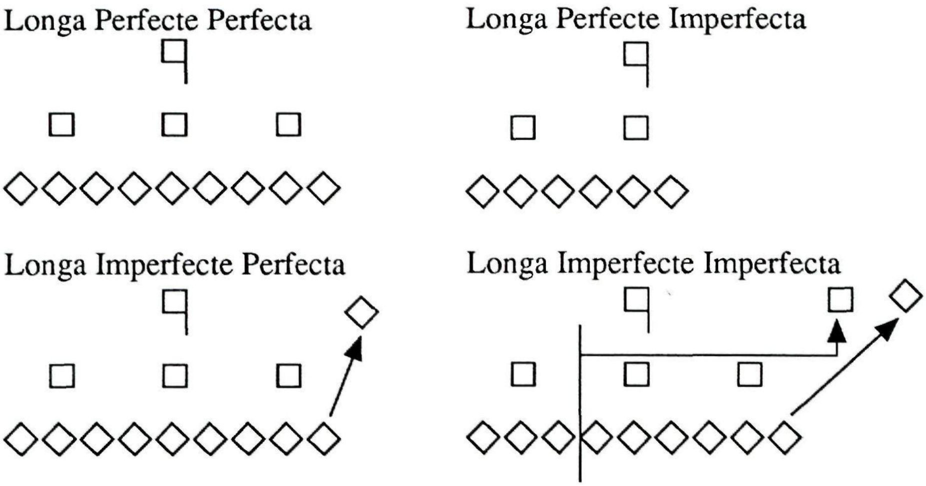

Figure 4

Composite proportions, then, serve to relate between the whole long and the parts of its parts. If so, are we not looking at the moment where music is no longer expected to reify those mathematical formulae that the Pythagorean mathematical tradition labelled perfect, simple or regular? We cannot avoid, then, a comparison between Jehan's use of proportions of proportions, and Bradwardine's mathematical theory of proportions of proportions as applied to the physical problem of determining the relation between velocity and forces.[26] As a consequence, physical processes of change are now accounted for by a new

[25] Johannes de Muris, *Notitia* (n. 17), p. 93: "Semibrevis est pars ternaria longae, nam est tertia pars brevis, et brevis est tertia pars longae, ergo semibrevis est tertia pars tertiae partis longae. Igitur imperficere potest longam."

[26] See above (n. 5).

type of mathematics – less simple, less "beautiful" but more articulate.[27]

This understanding becomes even more meaningful when we realize that the Pythagorean value system has a power of tenacity in music even when its conceptual base is being undermined. Jehan does not appear to realize fully the revolutionary potential inherent in his new perspective. He remains within the axiological climate of Pythagorean ideas, which had stated for generations the distinction between odd numbers, as associated with order and rationality, versus even numbers which since the Pythagoreans have symbolized boundlessness, disorder and irrationality. Jehan says: "Since the ternary number is found in all things in one form or another, its perfection here should not be doubted any more. And conversely, the binary number owing to its being shorter, remains imperfect, for the binary number is ill-famed."[28]

Jehan's predilection for ternary number has theological aspects as well. Time and again he repeats the old Christian belief that the Trinity – the principle of the world order – is reflected in all "perfections" throughout the chain of being:

> That all perfection rests in the ternary number is made clear by several likely conjectures. In God, the most perfect being, there is unity of substance, a Trinity of persons, threefold as one, one as threefold. There is a maximal correspondence of unity to Trinity. After God, in the separate intelligences, in the being and essence and in a composition of both (in one substance), the ternary number appears again. In the first celestial bodies [there are]: the mover, the moving, and time. Three (attributes) appear in the stars and the sun: heat, ray, splendor; in the elements: action, passion, matter; in individuals: generation, corruption, dissolution; in all finite time: beginning, middle, end; in all curable diseases: increase, stationary stage, decline.[29]

To sum up, Jehan's notion of correctness in music remained traditional and

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[27] As already mentioned, Thomas Bradwardine's solution presents a mathematical alternative that eliminates the difficulty involved in Aristotle's theory, see n. 6 above. His solution is incorrect in physics, yet it holds rue for music: since in order to double the velocity one raises the rhythmic values exponentially. Let me repeat at this point that although Bradwardine did not refer to contemporaneous music theorists, he mentioned the Pythagorean theory of harmony in conjunction with the legitimate use of quantitative methods for comparing qualitative differences. See Crosby (n. 5), *Thomas of Bradwardine*.

[28] Johannes de Muris, *Notitia* (n. 17), pp. 68-69: "Cum igitur ternarius omnibus se ingerat quodammodo, hunc esse perfectum non debet amplius dubitari. Per cuius oppositum, cum ab ipso recedat binarius, relinquitur imperfectus, cum etiam binarius numerus sit infamis."

[29] Johannes de Muris, *Notitia* (n. 17), p. 67: "Quod autem in ternario quiescat omnis perfectio, patet ex multis verisimilibus coniecturis. In Deo enim, qui perfectissimus est, unitas est in substantia, trinitas in personis; est igitur trinus unus et unus trinus. Maxima ergo convenientia est unitatis ad trinitatem. In intelligentia post Deum esse et essentia et compositum ex hiis sub numero ternario reperitur. In primo corporum caelo: movens, mobile, tempus. Tria sunt in stellis et sole: calor, radius, splendor; in elementis: actio, passio, materia; in individuis: generatio, corruptio, subiectum; in omni tempore finibili: principium, medium, finis; in omni morbo curabili: augmentum, status, declinatio."

anchored to the Pythagorean hierarchy of good and evil numbers. This is an insight of great importance for the understanding of the multi-layeredness of this crucial moment in music history. On the axiological level, age-old notions of musical quintessence are more powerful than their theoretical legitimation, and Jehan adheres to well-established Pythagorean norms while he fashions the conceptual ground for a new musical norm.

One manifestation of this new norm is the controversial issue of duple time, that is, of imperfection. To begin with, Jehan did not permit duple values as such. To understand Jehan's approach to imperfection in music, we need to consider the broader context of vigorous attempts in mathematics to mediate between difformity and uniformity, disorder and order, infinity and finality. Departuring from Gerard of Brussels' *Liber de motu*, fourteenth-century mathematicians searched for ways of translating non-uniform motions, that is, translating motions of changing velocity into a simple and uniform motion.[30] This procedure – to recall – involves the possibility of finding a simple uniform state or motion equivalent to a difform and irregular state or motion. Perhaps the most famous result of such endeavors to mediate between opposites was the formulation of the rule of "mean speed," which translated a uniformly difform motion into a simple uniform motion: According to this rule "A body moving uniformly difformly (in uniform acceleration) will in a given time cover the same distance it would while moving uniformly with its mean speed."[31] This basic rule engendered a list of variations that need not concern us here. What makes such transformations of crucial significance for the history of ideas is their dissociation from the Aristotelian scheme of conceptualization by the principle of

[30] Motion with changing velocity or difformly qualified subject implied the notion of infinity. Motion, itself continuous, was measured against the continuum of time, thus involving the notion of infinite parts that compose a continuum, and hence representing the infinite number of different velocities that compose a uniformly accelerated motion.

[31] Clagett (n. 23), *Science of Mechanics in the Middle Ages*, pp. 244-246: "Cum motus localis uniformiter difformis correspondet quo ad effectum suo medio gradui, sic patet quod tantum per idem tempus ponitur pertransiti per medium gradum sicut per illum motum localem uniformiter difformem." The quantification of the relation between disorder and order included other types of changes of motion, such as: "If a subject has a given degree at one end and increases rapidly in temperature at first as one moves away from this end of the subject, and then, increases more and more slowly with distance until the degree of the other end is reached, it will correspond to a greater degree than a subject whose temperature increases uniformly from one end to the other." (Sylla (n. 15), "Medieval Concepts," p. 257.) Other equations could involve two different types of motions, circular and linear. The following rule concerns the problem of rotation in which the speed of each point of the rotating body is in direct proportion to the distance of the point from the center of the body: in a uniform motion of a rotating wheel, each point of which is moving with different velocity, the velocity of the wheel as a whole is measured by the linear path traversed by the point which is in most rapid motion. See Clagett (n. 23), *Science of Mechanics in the Middle Ages*, pp. 235-237. Again, a motion composed of infinite velocities, represented by the infinite number of points on the radius of a rotating wheel, is translated into a simple uniform motion.

contrarieties. Mathematicians of the fourteenth century who worked with the mathematical concept of a continuum demonstrated an affinity, hitherto inconceivable, between the poles of order. Thus difformity ceases to designate the inherent evil of contingencies. The irrational, the infinite and the difform were redeemed.

Curiously, Jehan used analogous procedures to deal with the problem of imperfect, binary values, which he was not ready yet to accept as a principle. He believed that he needed to demonstrate that rhythmic imperfection, or rhythmic disorder, could be translated into terms of perfection and order. He observed that the three imperfect binary numbers of his four grades of perfection (Fig. 3), namely, numbers 6, 18, 54, are also multiples of 3. Hence they resemble ternary numbers and participate in the Holy Trinity.

$$6 = 2 \times 3$$
$$18 = 2 \times 3 \times 3$$
$$54 = 2 \times 3 \times 3 \times 3$$

In this way the concept of perfection in rhythm was broadened to include numbers that are partly composed of the number 3 and its powers. In music, as in mathematics, order and disorder were no longer mutually exclusive concepts. Jehan says:

> Since the ternary number is found in all things in some form or another, it may no longer be doubted that it is perfect. And conversely the binary number, since it falls short of the ternary, also since it is thus of lower rank, is left imperfect. But any composite number formed from these may properly be considered perfect because of its resemblance and agreement with the ternary number.[32]

[32] Johannes de Muris, *Notitia* (n. 17), pp. 68-69: "Cum igitur ternarius omnibus se ingerat quodammodo, hunc esse perfectum non debet amplius dubitari. Per cuius oppositum, cum ab ipso recedat binarius, relinquitur imperfectus, cum etiam binarius numerus sit infamis. Sed unum compositus, sic quilibet numerus convenientiaque, quam habet ad ternarium, perfectum potest merito reputari." The proposed reading of the above-quoted passage differs from that of Ulrich Michels; according to Michels, the sentence, "Sed unum compositus sic quilibet numerus convenientiaque, quam habet ad ternarium, perfectum potest merito reputari" refers to the unit, i.e., "compositus" is (for Michels) the unit of each *gradus perfectionis*. See Michels (n. 17), *Corpus Scriptorum*, p. 76. This interpretation conflicts with Jehan's theory and also with Michels's own commentary. For Jehan, all the values functioning as the unit of each grade had a double meaning, being both the neutral unit of the next greater ternary (or perfect value) and the perfect value of the next smaller value, as Jehan indicated clearly in his table represented in Fig. 3. The point to be stressed is that Jehan did not need any proof for the perfection of these unit-values because they are, at one and the same time, ternary and unity, therefore, perfect numbers. His main goal was to account for the use of imperfect values in the *ars nova* of the fourteenth century.

Speculations secundum imaginationem

The repercussion of the above principle of mediations between conceptual poles can be further put into relief within the broader cultural context of the fourteenth century. Despite the hazards of venturing into observation on the dominant issue of a culture, the all-pervasive preoccupation with the scope and limit of the notion of possibility in the culture of the fourteenth century cannot be disregarded. It extends into the domains of logic, theology, physics and mathematics, thereby becoming a truly obsessive, epoch-making issue. How far can one go with one's imaginative faculties? How far can logical inferences be excogitated as merely logically valid, yet physically fantastic? How far can God himself go in his thoughts or actions? And – if one extends the issue of possibility to music theory and to music practice – what are the limits of rhythmical possibilities? The second book of Jehan's *Notitia*, entitled *Musica practica* is clearly about such possibilities and not merely about actual, historical musical facts. The very organization of the book is quite telling in this respect. The *Notitia* seems to follow an Euclidian model. In other words, from the level of strictly delimited assumptions concerning primary issues, there derives the largest number of permissible conclusions. Jehan's nine conclusions (Fig. 5) focus on a single issue: the possibility of imperfecting rhythmic values. His strategy of narrowing the scope of problems under scrutiny, while exploring in depth one single issue, calls to mind contemporaneous mathematical works. Mathematical treatises, like Jehan's *Notitia*, begin with *notandae* and postulates, followed by an exhaustive list of conclusions. These conclusions push to the extreme the scope of variant cases, implied, even if remotely so, by the original problem and its rule.[33] Far-reaching conclusions could border on the absurd; but notwithstanding their fantastic content, they were logically valid inferences. Jehan's treatise adheres to this format. His conclusions (Fig. 5) begin with traditional simple cases of imperfection (conclusions 1-3) and move to new and rather intricate cases of imperfection. Indeed, these cases – the fruit of an imaginary speculation – epitomize the precepts of arguing *secundum imaginationem*. No musical counterparts seem to exist in the first half of the fourteenth century. That is to say that it is the logical validity of the derivations that calls for these conclusions and not their actual realization. Jehan conceives his nine

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[33] See J. Murdoch, "The Development of a Critical Temper: New Approaches and Modes of Analysis in Fourteenth-Century Philosophy, Science, and Theology," in S. Wenzel (ed.), *Medieval and Renaissance Studies*, vol. 7: *Proceedings of the Southeastern Institute of Medieval and Renaissance Studies* (1978), pp. 56-57. See also J. Murdoch, "Mathesis in Philosophiam Scholasticam Introducta. The Rise and Development of the Application of Mathematics in Fourteenth Century Philosophy and Theology," in *Arts libéraux et philosophie au Moyen Age: Actes du Quatrième Congrès International de Philosophie Mediévale* (Montréal: Institut d'Études Mediévales, 1969), pp. 225-238.

conclusions as the latent secrets of the musical art, implied by its basic rules and given within the limits set by the ancients.[34] Furthermore, he argues that his conclusions represent an incomplete list of rhythmic possibilities; many other conclusions are implicit and may be discovered through further exercises.[35]

1. A long can be imperfected by a breve
2. A breve can be imperfected by a semibreve
3. A semibreve can be imperfected by a minim
4. A long can be imperfected by a semibreve
5. A breve can be imperfected by a minim
6. A minim cannot be imperfected
7. An altered breve can be imperfected by a semibreve
8. An altered semibreve can be imperfected by a minim
9. Time can be divided into as many equal parts as one wishes

Figure 5

Jehan, then, looks beyond the musical practice, thereby preconceiving rhythmical progressions that would appear in actual musical compositions several decades later. This is not a self-evident procedure. It seems to call for some kind of explanation that would situate it within a general conceptual framework, where the derivations of logical possibilities take priority over the listing of empirical data. The immediate frame of reference which comes to mind is that of fourteenth-century theology, where the notion of infinite possibilities is highly elaborated. Much has been said on fourteenth- century science as having been motivated, or perhaps safeguarded, by the maxim that God, by virtue of his absolute power (*de Potentia Dei absoluta*), can do anything that does not involve a contradiction.[36] This gave a licence to expand investigations beyond the paradigmatic principles that had governed traditional bodies of knowledge. It not only encouraged open-mindedness, but called explicitly for variations and innovations. The principle of God's absolute power guided scholars of the

[34] Johannes de Muris, *Notitia* (n. 17), p. 85: "In arte nostra hac inclusa sunt aliqua quasi abscondita intus latentia ... Nec insurgat invidus res; rehensor, si qua dicere cogamur inaudita modos vocis apparentiaque salvantes insequendo semper limites antiquorum."

[35] *Ibid.*, p. 106: "Sub istis novem conclusionibus declaratis multae latent conclusiones aliae speciales, quae per exercitium erunt studentibus manifestae."

[36] See Murdoch (n. 33), "The Development of a Critical Temper," pp. 53-55.

fourteenth century away from the realm of physical possibilities to the infinitely more complex and imaginary space of logical possibilities, so impatiently explored by the Mertonian Calculators.[37]

In this context, Jehan's *Notitia*, specifically its organization and its speculative variant cases of rhythmic imperfections finds its home-ground. In other words, it is the issue of God's infinity that induces the fabrication and examination of possible worlds, permeating among other fields of discourse, the seemingly local field of rhythmic notation.

Granted that some of Jehan's conclusions designate possibilities that did not exist then in actual music, in themselves those conclusions present yet another singularity. In fact, they are contradictory: the sixth and the ninth conclusions do not cohere. According to the sixth conclusion, the rhythmic minim is a discrete and indivisible unit. According to the ninth conclusion, time is a continuum divisible to infinity and there is no absolute discrete minim at all. How can musical time be both subjected to a confined system of constraints, yet at the same time be indefinitely divisible? How can a minim be a final limit while time is infinitely divisible at will?

A deeper meaning than a mere passing incompetence on the part of Jehan may be extracted from this contradiction. For example, one may understand the sixth conclusion as referring to the physical limit of the minim, that is, taking into account that there is a limit to the shortest length of sound. Alternatively, we could understand the notion of a minim as an *ad hoc* designation of the smallest constituent of a rhythmic whole. If so, the sixth conclusion would indicate the fact of discreteness, which can always be redefined because of the continuity of time and its indefinite divisibility.[38] Thus, Jehan generalizes the possible divisions of a rhythmic whole, while conserving, rather than breaching, the very foundation of his own system. In essence, he is bridging between two classes of rhythmic units – the discrete units of arithmetics and the continuous units of geometry. In principle, a rhythmic whole can be divided now into its arithmetical units and, at the same time, into its geometrical units. This is a new understanding of the notion of a rhythmic part, and a new conception of whole-to-part relationship in music. Jehan therefore broadens the scope of axioms by extending their validity to include not only the realm of discrete magnitudes, but also that of continuous magnitudes.

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[37] *Ibid.*, p. 53.

[38] The conceptualization of the rhythmic minim demanded awareness of the crucial distinction between a mathematical sizeless atom and a physical atom, which has a certain minimal extension. Furthermore, it required insight into the subtle relation between language and reality. Jehan's theory exhibits sensibility to both demands. For the period's struggle to grasp the concept of a rhythmic minim, see Tanay (n. 2), *Music in the Age of Ockham*, pp. 88-94.

Ontological broadening in music and mathematics: towards a new concept of beauty

Jehan's theory can be related to the very core of the mathematical work of Nicole Oresme. Oresme, Jehan's younger contemporary, was born around 1335 and died in 1382. According to E. Grant,[39] Oresme may have indeed been indebted to Jehan's mathematical works: a comparison between sections from book IV of Jehan's *Quadripartitum numerorum* and Oresme's treatises (*Ad pauca respicientes* and *Tractatus de commensurabilitate vel incommensurabilitate motuum celi*) exhibits similar and sometimes nearly identical propositions.[40] Grant, however, distinguishes between the question of determining chronological precedence and the problem of proving direct quotation, or even awareness, and concludes that regarding both questions no definitive answer can be proposed. The main difficulty is the inconclusiveness of the evidence itself. Although in one copy of Oresme's treatise, *On ratios*, we are told that Oresme used Jehan's text, we know neither the identity of the said text, nor the date of this particular copy of Oresme's treatise.[41] Furthermore, we have no information about the scribe and his reliability.[42] Last but not least, we need to evaluate the scribe's testimony against Oresme's own claim in his *De commensurabilitate* that his work is original.[43] That Oresme was profoundly interested in music is evident in several different treatises, including, inter alia, the said *De commensurabilitate* and *De configurationibus qualitatum et motuum*. A complete study of Oresme's musical thought is still waiting for further studies, and here I wish only to suggest the general layout of the hitherto unnoticed accordance between Jehan's new theory of whole-to-part relationship in music and Oresme's mathematical and astronomical theories of the relation between a whole and its parts. To put it schematically, Oresme departs from the Euclidian model and broadens the mathematical conception of whole-to-parts relationship. He expands the notion of commensurability between proportions, to include not only rational proportions of finite magnitudes, but also irrational ones. Euclid's theory of commensurability was confined to rational proportions that could be related as whole

[39] E. Grant (ed.), *Nicole Oresme and the Kinematics of Circular Motion: Tractatus de commensurabilitate vel incommensurabilitate motuum celi* (Madison, Wisconsin: Wisconsin University Press, 1971).

[40] *Ibid.*, pp. 86-101.

[41] *Ibid.*, pp. 98-99.

[42] E. Grant's main goal is to establish and prove the originality of Oresme's theory on the incommensurability of celestial motions; he therefore assumes the precedence of Jehan's *Quadripartitum* of which Oresme had read the relevant chapters, in order to show that Oresme's theory "far surpassed in extent, subtlety, and content the few chapters in the *Quadripartitum*." *Ibid.*, p. 101.

[43] *Ibidem*.

and parts by a common denominator that was an integer. Nicole Oresme, however, proves that the axioms of proportionality, or commensurability, between finite magnitudes could be properly applied in cases of infinite magnitudes. Proposing such a new understanding of the notion of part in mathematics, Oresme relates irrational proportions and rational ones by demonstrating the commensurability of irrational proportions to rational ones, as illustrated in Fig. 6.1. Oresme further extends his use of the term 'common part', proving that an irrational proportion can be commensurable with another irrational proportion (Fig. 6.2).

$$4/1=(4/1)^{1/4}\bullet(4/1)^{3/4}$$

Figure 6.1

$$8^{\sqrt{2}}/1=\left[2^{\sqrt{2}}/1\right]^{3/1}$$

Figure 6.2

To be sure, Oresme's mathematics is much more sophisticated and far reaching than Jehan's rhythmic innovation. Jehan confines his discussion solely to the possibility of dividing a given unit of time, taken as a continuum, into as many equal parts as one wishes, up to nine equal parts. Yet both focus on the ontological expansion of traditional fields of knowledge, and this interest in the idea of multitude in itself opens new horizons for composers and performers.

Jehan is fascinated by the possibility of diversifying the rhythms by shifting between various divisions of given note-values, and endows a certain degree of *bravura* to those artists capable of accomplishing such rhythmic diversity. For him, "laudable and masterful will be a musician who would make music by dividing the same segment of time, now into two equal parts, now into three and into all the other possible equal parts."[44] Between these lines one can read a tacit acknowledgement of the esthetic value of variation and multiplicity. Significantly, Nicole Oresme expresses a similar ideal in his remarks on musical

[44] Johannes de Muris, *Notitia* (n. 17), p. 105: "Laudabilis autem esset musicus et peritus, qui super idem tempus aequale ipsum dividendo nunc per duas, nunc per tres et ceteras partes integre discantaret."

esthetics.[45] It is first and foremost Oresme's conception of beauty in the world and its mathematical base that loads his esthetics of music with such a powerful potential for the future. For Oresme, beauty in the world implies the presence of imperfection and multiformity; variations, and disturbances are therefore parts inherent in the perfection of the totality.[46] This statement has two major aspects, one in the field of pragmatic esthetics, the other in Oresme's mathematics and cosmology. Oresme insists on the importance of novelty as a criterion of esthetic value. Novelty is only possible where variability and multiplicity are recognized as legitimate and constitutive dimensions of the whole. Consequently, it is the principle of novelty that contains the promise for a rewarding musical reception for Oresme. He asks: "What song would please that is frequently or oft repeated? Would not such uniformity (and repetition) produce disgust? It surely would, for novelty is more delightful. A singer who is unable to vary musical sounds, which are infinitely variable, would no longer be thought best, but (would be taken for) a cuckoo."[47]

Oresme's demands recall Jehan's appreciation of masterful musicians who can diversify their music by alternating artfully between various rhythmic combinations. Oresme, however, does not go into detail as to how musical practices are in fact to achieve this rewarding experience.[48] It is rather the conceptual substructure on which the whole legitimation of novelty rests that makes Oresme relevant to Jehan's *ars nova* and to the future of music in such an important way. Through Oresme's handling of the cosmological notion of variability and its relationship to incommensurable proportions, a new perspec-

[45] See V. Zoubov, "Nicole Oresme et la Musique," *Medieval and Renaissance Studies* 5, 1961, pp. 96-107. This is the only study of Oresme's musical esthetics within the broader context of his general world view. Zoubov actually notes that the notion of novelty - as accentuated in Oresme's writing - recalls the title of Jehan de Meur's *Ars nova musicae* (better known in current literature as *Notitia artis musicae*) and Philippe de Vitry's *Ars Nova*. But he does not pursue in detail the affinity between Jehan's and Oresme's thought. For the relation between Oresme's *Geometry of Qualities and Motions* and the contemporaneous musical theory and practice see Tanay (n. 2), *Music in the Age of Ockham*, pp. 193-204.

[46] Grant (n. 39), *Nicole Oresme*, pp. 312-313: "Et celum insignius, quam si essent stelle ubique per totum; ymo universum perfectius est propter corruptibilia et etiam propter imperfecta et monstra."

[47] *Ibid.*, pp. 316-317: "Que est ista cantilena que placeret sepe aut multotiens repetita? Nonne talis uniformitas gignit fastidium? Ymo certe, et novitas plus delectat. Nec esset reputatus cantor optimus sed cuculus, qui non posset modulos musicos variare qui sunt variabiles in infinitum."

[48] V. Zoubov had already noted the discrepancy between Oresme's revolutionary cosmology, on the one hand, and his traditional approach to the musical consonances and dissonances, on the other, see Zoubov (n. 45), "Nicole Oresme et la Musique," pp. 105-107. Furthermore, Oresme did not refer to contemporaneous music or theories of music. This does not render his esthetical-musical doctrine insignificant, especially in light of the accordance between his insight and that of Jehan de Meur.

tive of possible musical procedures becomes feasible in principle, though naturally at the time it could not be recognized for what it was. Oresme proves that some celestial motions are incommensurable with one another and, therefore, celestial constellations "should not be repeated so often, but that (on the contrary) new and dissimilar configurations should emerge from previous ones and always produce different effects."[49] The mathematical model that corresponds to this world view consists of the assumption that with any random number of rational proportions, forming one or more series, those that are mutually commensurable are much fewer than those that are incommensurable. As a result, it is likely that any two proposed unknown proportions (now representing, according to Oresme, two celestial motions) are incommensurable.[50]

In Oresme's view, a carefully balanced mixture of incommensurable and commensurable celestial motions makes the universe more beautiful and more graceful:

> However, the heavens would glitter with even greater splendor if the bodies were commensurable and their motions incommensurable, or if some motions were commensurable and other incommensurable, where all are regular (and uniform), than if all were commensurable. By mixing together irrationality and regularity, the regularity would be varied by the irrationality, and the irrationality, with regularity bound to it, would not be deprived.[51]

By analogy, Oresme encourages variability in worldly artistic production:

> A song with its consonances varied is sweeter than if it were constituted continually from the best consonance (that was unvaried), namely, a diapason; and a picture decorated with different colors is more beautiful than one in which the most beautiful color is spread uniformly over the entire surface.[52]

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[49] Grant (n. 39), *Nicole Oresme*, pp. 316-317: "quod non totiens repetatur idem sed quod novas et dissimiles prioribus constelationes affectusque varios semper producat."

[50] Nicole Oresme's proposition on mathematical probability appears in E. Grant (ed.), *Nicole Oresme: De proportionibus proportionum and Ad pauca respicientes* (Madison, Wisconsin: Wisconsin University Press, 1966), pp. 247-255.

[51] Grant (n. 39), *Nicole Oresme*, pp. 310-311: "Verumtamen celestia multo ampliori fulgent decore si corpora sint commensurabilia et motus incommensurabiles; aut si aliqui motus sint commensurabiles et alii incommensurabiles qui omnes sunt regulares quam si cuncta essent commensurabilia, ut scilicet irrationalitate et regularitate commixtis, regularitas irrationalitate varietur, et irrationalitas regularitate debita non fraudetur."

[52] *Ibid.*, pp. 312-313: "Cantusque consonantiis variatus dulcior quam si fieret continue optima consonantia scilicet dyapason; et pictura variis distincta coloribus speciosior colore pulcherrimo in tota superficie uniforiter diffuso."

In the new astronomy and esthetics, irrational proportions are not taken as some kind of evil or terrifying *terra incognita*, but are assimilated into the very idea of a perfect whole.[53] Here we arrive at the very core of the shift in the relation between mathematics and physics. Oresme's cosmology does not reify those mathematical formulae that the Pythagorean tradition considered as perfect and simple. It is a cosmology founded on a new type of mathematics, which denies qualitative distinctions within mathematics. In Oresme's world view the factor of indeterminacy becomes dominant. It undermines the Greek and Roman beliefs that the celestial motions were related by rational ratio. It goes against the very grain of traditional doctrines of the general uniformity of nature, as exemplified by the regular repetitions of celestial configurations and events. It was this outlook that perpetuated for generations the Pythagorean *harmonia mundi* as integral to the principle of musical concordances. As already mentioned, Oresme sees the esthetic principle of variability as related, on the one hand, to his notion of beauty and, on the other, to his tacit ground for the legitimation of novelty as an esthetic value. In his time, there was no comprehensive esthetic doctrine in existence that could have profited from this insight and carried it on. Taking a historical perspective, it seems rather esoteric. Yet when we consider Oresme's latent esthetics in the framework of both Jehan's theory of the *ars nova* and some of the actual developments in music in the second half of the fourteenth century, we see a common conceptual structure. Jehan conceives rhythmic totality to be composed of perfect as well as imperfect relations. He searches for the limit of rhythmic diversion. His theory adumbrates musical tendencies that are crystallized only later on, in the fourteenth century, when the musical process is variegated by the mixture of rhythmic units that are measured by the multiplications of the indivisible minim and, at the same time, by new units, produced by the division of that minim into its continuous parts. It is by way of dealing with these minute units that we see a new and interesting application of the traditional Pythagorean harmonic proportions, apart from and in conflict with their traditional function in musical as well as cosmological theories. In the music of the late fourteenth century we see that the mathematical aspect of musical consonances is transferred from the realm of static harmony to the realm of dynamical processes involved in rhythmic progressions – namely, to the cause of measuring rate of changes, that is, diminutions of rhythmic units. Theorists and composers treated the seemingly paradoxical notions of values smaller than the minim by a special rhythmic technique called rhythmic proportions. This procedure made it possible to diminish the standard minim in a certain rational proportion. Proportions defined the numerical relation between

[53] *Ibid.*: "Nam sive irrationalis proportio sit nobilior sive non, earum tamen congrua commixtio pulchrior est singularitate uniformi."

the various particles of the minim. A given proportion indicated the substitution of a group of standard minims by a new and rather denser group of minims, yet equipollent as a whole to the original standard group. The new group of minims diminished the standard group of minims in the given proportion. For example in the proportion of 4:3, four new minims equal three standard minims. Theorists were especially fascinated by the possibility of diminishing the rhythmic minim by the proportions of *harmonia mundi*, that is 2:1, 3:2, 4:3, and 9:8. But in some extreme cases the standard minim could also be diminished by other non-harmonic proportions such as 5:2, 7:2, 8:3, and 10:3.[54]

In conclusion, by the late fourteenth century the old Pythagorean harmonic proportions have become mere mathematical signs representing the rate of change in the rhythmic motion. Somewhat less privileged, harmonic proportion functions hereafter side by side with non-harmonic proportion, for the sake of measuring the earthly and imperfect property of variability in the rhythmic motion. This forms the conceptual nucleus of a future shift from the understanding of music as a mirror of a rigid transcendent or mathematical order to the understanding of music as a flexible language attuned to the flexibility of phenomenal contingencies. Earlier in my paper I suggested that discussion of conceptual homologies and structural correspondences may shed new light on the development of a new concept of music, namely, the evolution of the notion of *ut oratoria musica*. This notion sees music as a form of poetic eloquence, concomitant to the emergence of a musical dynamic harmony. It is the multiform search for mediation between perfection and imperfection, the very alertness to the difformities and variabilities that emerges in structural relation to new possibilities in music. And the new fourteenth-century idea of beauty that includes its own disturbances could have prepared the ground, if not the very conceptual premises, for the early modern notion of harmony as a bond of consonances and dissonances.

While a narrow historical explanation cannot be claimed for such a dramatic and crucial development, one cannot ignore the substantiality of the role played here by the newly elaborated mathematical substructure. This is not to say that other factors must be regarded as marginal. The dense texture of historical development must allow for some opacities. We are dealing here with no less than the germinal phase of the emergence of a new esthetic norm that has only gradually come to the fore, reaching its mature conceptualization generations later.

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[54] For the origin and development of proportions in the fourteenth century theory and practice, see A. Busse-Berger, "The Origin and Early History of Proportion Signs," *Journal of the American Musicological Society* 41, 1988, pp. 403-433. For the use of proportion in late fourteenth-century music, see J. Hirshberg, *The Music of the Late Fourteenth Century: A Study in Musical Style* (Doctoral Dissertation, University of Pennsylvania, 1971), pp. 332-352.

Summary

This paper attempts to explore the relation between the revolutionary musical theory of Jehan de Meur and the contemporaneous mathematical works of the Mertonian Calculatory tradition. It argues that the innovative Mertonian theory of quantifying intensive changes had an unexpected parallel in Jehan de Meur's new theory of rhythmical notation as expounded in his Notitia artis musicae of 1321. It demonstrates further that in music as in mathematics and philosophy the fourteenth century saw the evolution of a new attitude towards irrationality and disorder, that provided the nucleus to the crucial shift from the Medieval notion of music as an image of divine perfection to the early modern notion of music as a dynamic process attuned to human's affections, fears and desires.

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