"ALL PHENOMENA OF LIGHT THAT DEPEND ON MATHEMATICS"

A SKETCH OF THE DEVELOPMENT OF NINETEENTH-CENTURY GEOMETRICAL OPTICS[1]

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Introduction

In recent years, the history of nineteenth-century optics has received considerable attention. The studies of Fox, Silliman and Buchwald have made it clear that nineteenth-century optics was a very rich field which in many ways was central to the canon of nineteenth-century science. Accordingly, the history of the field offers valuable insights into the dynamics of nineteenth-century science in general. However, many of the more recent studies on nineteenth-century optics concentrate on physical optics, i.e., the theory of the nature of light. In particular the rise of physical optics at the beginning of the century as a clearly delineated field has been extensively studied.[2] In contrast, the history of the part of optics known as geometrical optics, i.e., the mathematical study of light which does not take its physical properties into account, has received very little attention.

In the context of the nineteenth century, geometrical optics comprises all studies which were concerned with the description of the phenomena of light in terms of rectilinear light rays. The literature on the history of the field, however,

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[1] This paper is a revised and expanded version of the first chapter of my Ph.D. thesis, *The Structure of Systems of Lines in 19th Century Geometrical Optics: Malus' Theorem and the Description of the Infinitely Thin Pencil* (Utrecht, 1993). I would like to thank here my supervisors Professor H.J.M. Bos and Dr C. Hakfoort for their comments on this chapter. I would also like to thank the editorial board of *Tractrix* for its comments on an earlier version of this paper.

[2] Cf. Robert Fox, "The Rise and Fall of Laplacian Physics," *Historical Studies in the Physical Sciences* 4, 1974, pp. 89-136; Robert Silliman, "Fresnel and the Emergence of Physics as a Discipline," *Ibid.*, pp. 137-162; Jed Z. Buchwald, *The Rise of the Wave Theory of Light. Optical Theory and Experiment in the Early Nineteenth Century* (Chicago, 1989) and the literature mentioned there.

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is heavily biased towards the history of technical optics, i.e., the theory of the construction of optical instruments. Most of this literature was written in the course of the twentieth century. The scientists employed at Zeiss in Jena were particularly interested in the origins of this branch of geometrical optics.[3] In the history of the other branches of geometrical optics and especially in the more mathematical parts of the field, there was little interest. In the early twentieth century, a chapter on geometrical optics and the theory of optical instruments was projected for the *Encyklopädie der Mathematischen Wissenschaften*. In the end, however, this chapter never appeared, because no suitable collaborators could be found.[4] After the Second World War, even the interest in the history of nineteenth-century technical optics subsided. Although the history of the construction of optical instruments continues to be studied, the emphasis is mostly on practical aspects, such as the production of glass for the lenses or the production of the mechanical parts of optical instruments, or on the instrument makers themselves.[5]

This lack of attention to the history of geometrical optics as a whole can be largely explained by the position of the field compared to that of physical optics. Whereas from the first decades of the nineteenth century onwards, physical optics had developed into a field of sufficient independence to warrant historical investigations on its own account, geometrical optics never achieved such a status. Since the beginning of the nineteenth century, it was always a field in between other fields. On the one hand, during much of the nineteenth century, geometrical optics was very close to mathematics and its problematics was much

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[3] Cf. Max Herzberger, "Geschichtliche Bemerkungen," in M. Herzberger, *Strahlenoptik* (Berlin, 1931), pp. 179-190 and "Geschichtlicher Abriß der Strahlenoptik," *Zeitschrift für Instrumentenkunde* 52, 1932, pp. 429-435, 485-493, 534-542. Herzberger was one of the principal opticians at Zeiss in the early 1930s. Furthermore, there was the periodical *Forschungen zur Geschichte der Optik*, which appeared as a supplement to the *Zeitschrift für Instrumentenkunde* in the 1930s and was edited by Moritz von Rohr, one of Herzberger's superiors at Zeiss. In this periodical, quite a number of papers on various aspects of the history of geometrical optics were published. Clearly, however, not only Herzberger but also the contributors to the *Forschungen* viewed geometrical optics as just a broad background to technical optics.

[4] See *Encyklopädie der Mathematischen Wissenschaften*, Bd. 5.3, p. 1215 (postscript by Sommerfeld of February, 1926).

[5] Cf. Dieter Herbst, "Das Wechselverhältnis von Astronomie und Machinenbau, dargestellt am Beispiel des Wirkens von Georg von Reichenbach," *Schriftenreihe für Geschichte der Naturwissenschaften, Technik und Medizin* 28, 1991, pp. 61-72; Myles Jackson, "Die britische Antwort auf Fraunhofer und die deutsche Hegemonie in der Optik," in *Deutsches Museum Jahrbuch* (München, 1992). An exception is formed by Joachim Wittig, "Zur Disziplingenese der technischen Optik," *Studien zur Entstehungsgeschichte technikwissenschaftlicher Disziplinen. III. Rostocker Wissenschaftshistorisches Symposium vom 2. bis 4. Dezember 1982* (Berlin, 1985), pp. 13-31. As its title already indicates, this papers concentrates on the periodisation of technical optics as a whole. See also Peter de Clercq (ed.), *19th Century Scientific Instruments and their Makers* (Amsterdam/Leiden, 1985).

influenced by the development of mathematics. On the other hand, the development of geometrical optics was also strongly influenced by the exigencies of those who sought to apply geometrical optics to the construction of optical instruments or to the study of the functioning of the eye. Any study on the history of geometrical optics has to take at least some of these influences into account. For this reason, the history of geometrical optics is difficult to investigate. At the same time, however, it is precisely because nineteenth-century geometrical optics occupied an intermediate position between mathematics and more practical fields that a detailed picture of its development would be of considerable interest. Such a study would provide an interesting example of how the different sciences of the nineteenth century interacted. More specifically, it would tie in with the recent interest in the relation between mathematics and the other sciences in the nineteenth century.[6] In this paper, I hope to provide a first step toward the construction of such a detailed picture by sketching the history of nineteenth-century geometrical optics and placing its main developments in the context of the history of mathematics, of optics at large and of the construction of optical instruments.[7]

The period covered in this paper begins in 1808 and ends a century later, somewhere around the time of the First World War. The first year has been taken because in this year the "Traité d'Optique" by the French physicist and mathematician Etienne Malus (1775-1812) appeared. In this paper a new, more abstract approach to geometrical optics was proposed which would allow to transcend the restrictions of eighteenth-century optics. The end of the period is chosen around World War I because only by then was the transformation of geometrical optics as prefigured by Malus effectuated.

Before the nineteenth century, a mathematical tradition in optics did exist, but this tradition was mainly concerned with the study of the formation of images in optical instruments. It is in this light that we have to consider Malus' "Traité". Basically, Malus proposed to extend this classical approach to the study of systems of light rays in homogeneous media. Although the importance of this extension of the classical tradition was recognised by some, this very general formulation of geometrical optics was certainly not generally appreciated. During

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[6] Cf. Erhard Scholz, *Symmetrie-Gruppe-Dualität. Zur Beziehung zwischen theoretischer Mathematik und Anwendungen in Kristallographie und Baustatik des 19. Jahrhunderts* (Basel, 1989) (= *Science Networks-Historical Studies*, vol. 1.); Herbert Mehrtens, *Moderne-Sprache-Mathematik. Eine Geschichte des Streits um die Grundlagen der Disziplin und des Subjekts formaler Systeme* (Frankfurt, 1990); Ivor Grattan-Guinness, *Convolutions in Mathematics*, 3 vols. (Basel, 1991) (= *Science Networks-Historical Studies*, vols. 2-5).

[7] A more detailed rendering of the more mathematical aspects of (some parts of) 19th century geometrical optics can be found in Atzema (n. 1), *The Structure of Systems of Lines*.

the first two or three decades of the nineteenth century, the influence of Malus' work on the development of geometrical optics was rather slight. The mathematical tradition in optics was still too much rooted in its practical origins for a more theoretical approach of the problems of geometrical optics to be considered worthwhile. Only after the 1840s was there a growing interest in a more abstract approach to geometrical optics. It was in this period that Malus' work began to be appreciated and that Malus himself acquired the status of harbinger of a new kind of geometrical optics. An important factor behind this belated endorsement of Malus' "Traité" was the development of two important but thusfar rather hybrid problem-complexes in the mathematical tradition in optics into well-defined sub-fields of geometrical optics. The first of these concerned the study of the optical properties of the eye, which problem-complex developed into a sub-field referred to as the *dioptrics of the eye*. The other problem-complex was to do with the construction of optical instruments. By the 1890s, this problem-complex had developed into a coherent field which was referred to as *technical optics*. Although these fields were very practically oriented, both relied on a theoretical apparatus which was essentially a mathematically coherent elaboration of a number of basic assumptions. In this context, there was room for the mathematical study of the basic assumptions of geometrical optics as a goal in itself. This serves to explain the interest in an abstract approach to geometrical optics which had become visible by the end of the nineteenth century.

By the turn of the century, the abstract approach to geometrical optics had even become dominant. Even within the fields of technical optics and the dioptrics of the eye there was a strong interest in an abstract all-embracing formulation of geometrical optics. Where Malus had tried to give a formulation of geometrical optics in the most advanced mathematical language available in his time, it was now tried to express the laws of the field in terms of the most advanced mathematics that was available by the end of the nineteenth century. In the wake of many physicists and mathematicians who attempted to ply the laws of physics to the laws of mathematics, astronomers like Heinrich Bruns and Karl Schwarzschild tried to embed geometrical optics in mathematics as well. Whereas Malus' attempt at the mathematisation of geometrical optics had been largely abortive, the attempts of Bruns and Schwarzschild and their likes were successful. In a relatively short period the foundations of geometrical optics were thoroughly reformulated. The field itself was extended to the study of the optics of arbitrary media. With this transformation of the field, a new approach had finally been effectuated, a century after Malus had first suggested how the restrictions of the eighteenth-century paradigm of the mathematical tradition in optics could be transcended.

In this paper, I give a sketch of the developments in geometrical optics as a whole, leading from Malus' failure to get abstract methods accepted into geometrical optics to Bruns' and Schwarzschild's success in doing so. Special attention will be paid to the reasons why Malus' approach was not successful, whereas Bruns' and Schwarzschild's was.

Geometrical optics before 1800

Before the nineteenth century, the field of optics embraced at least two rather different traditions. One of these had to do with the *natural philosophical* aspects of light, i.e., with theories on the nature of light. The most influential theories put forward in this context were the emission or particle theories connected with Newton's name and the medium or wave theories as formulated by Huygens, Euler and others.[8] The other tradition was more mathematical in nature. Generally speaking, it was concerned with tracing the path of rectilinear light rays after reflection or refraction; the study of perspective, however, also formed part of this tradition. The optics related to reflection was known as catoptrics, the optics of refraction as dioptrics. In practice, it was primarily dioptrics that was studied, usually in the context of the functioning of optical instruments. Essentially, seventeenth- and eighteenth-century dioptrics went back to Kepler and his ideas about the functioning of the telescope as expounded in the *Dioptrice* of 1611.[9] Assuming that for all the points near to its axis the telescope yields a perfect, unique image, Kepler could formulate a theory of the formation of (very small) images. In the course of the seventeenth and eighteenth century, this theory was developed further by various mathematicians. Apart from the theory of the formation of images, the most important topic that was studied during the eighteenth century was the theory of the achromatic lens, i.e., a lens for which no dispersion (or shifting of colours) occurs. In the course of the eighteenth century, Alexis-Claude Clairaut (1713-1765), Leonard Euler (1707-1783) and Samuel Klingenstierna (1698-1765) wrote seminal papers on this subject.[10] Another subject that was initiated was the study of aberrations, i.e.,

[8] See Casper Hakfoort, *Optics in the Age of Euler* (Cambridge, to appear).

[9] Johannes Kepler, *Dioptrice, seu demonstratio eorum quae visui & visibilibus propter conspicilla non ita pridem inventa accident* (Augsburg, 1611; reprint 1962). In 1904, von Rohr published an annotated German translation of this booklet as vol. 144 of Ostwald's series *Klassiker der Wissenschaften*.

[10] For 17th century optics, see Alan Shapiro, "The *Optical Lectures* and the foundations of the theory of optical imagery," in Mordechai Feingold (ed.), *Before Newton: The Life and Times of Isaac Barrow* (Cambridge, 1990), pp. 105-178. Emil Fellman, "Leonard Eulers Stellung in der Geschichte der Optik," in *Euleri Opera Omnia*, ser. 3, vol. 9 (Leipzig/Berlin, 1973), pp. 297-322 is a good introduction to the 18th century and the history of the achromatic lens.

the study of the deviations from Kepler's perfect images. The contributions to the latter topic, however, received very little attention.

`Few attempts were made to connect the natural philosophical and the mathematical traditions in optics. Most works on the theory of catoptrics and dioptrics did not include natural philosophical considerations. Typically, a treatise that purported to deal with the whole of optics contained one chapter on the nature of light and a completely independent one on catoptrics and dioptrics.[11] One of the few attempts at uniting the natural philosophical and the mathematical tradition in optics in one theory was Christiaan Huygens' *Traité de la Lumière* of 1690. Significantly, however, this book met with little acclaim. It was not until the end of the nineteenth century that Huygens' work began to be fully appreciated, at which time it could not exert any influence any more.[12] In the seventeenth and eighteenth century, the general opinion seems to have been that catoptrics and dioptrics were independent of any theory about the nature of light. As a rule, the basics of the mathematical tradition in optics were considered as laws that were empirically founded. The natural philosophical tradition in optics was often viewed as a completely different field.

From the eighteenth into the nineteenth century

At the beginning of the nineteenth century, the relation between the mathematical and the natural philosophical tradition quickly changed. Under the influence of the rise of exact experimentation at the end of the eighteenth century,[13] the attitude towards theorising about the nature of light underwent a radical alteration. With, among others, Etienne Malus (17775-1812),

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[11] Cf. Robert Smith, *A Compleat System of Opticks, in Four Books, viz. a Popular, a Mathematical, a Mechanical, and a Philosophical Treatise. To which are Added Remarks upon the Whole*, 2 vols. (Cambridge, 1738).

[12] Christiaan Huygens, *Traité de la Lumière, où sont expliquées les causes de ce qui luy arrive dans la reflexion et dans la refraction et particulièrement dans l'étrange refraction du cristal d'Islande. Avec un discours de la cause de la pesanteur* (Paris, 1690) (= *Oeuvres Complètes*, vol. 19 (1937), pp. 451-537). An indication for the belated appreciation of Huygens' theory is that the *Traité* was not reprinted until the 1880s, when it was reprinted twice within a period of five years. In 1885, the French text was republished in Leipzig with a (short) introduction in Latin by Wilhelm Borchard; in 1895, an annotated German translation by Eugen Lommel appeared as vol. 20 of Ostwald's series *Klassiker der Wissenschaften*. Of course, Fresnel had been interested in Huygens' *Traité* as well, but in many ways he seems to have been an exception. On the reception of the *Traité de la Lumière* during the 18th century, see Hakfoort (n. 8), *Optics in the Age of Euler*.

[13] For a very suggestive account of this process, see Thomas S. Kuhn, "Mathematical vs. Experimental Traditions in the Development of Physical Science," *Journal of Interdisciplinary History* 7, 1976, pp. 1-31 (= Kuhn, *The Essential Tension* (Chicago, 1977), pp. 31-65).

Jean-Baptiste Biot (1774-1862), Dominique-François Arago (1786-1853) and, later on, Augustin Fresnel (1788-1827), the natural philosophy of optics took another direction. As before, the principal issue was the question of whether light consisted of particles or should be viewed as waves moving in ethereal matter. In marked contrast to the approach during the seventeenth and eigtheenth century, however, attempts were made to confirm or invalidate the two theories of light by means of carefully designed experiments.[14]

If these experiments were to be conclusive, both theories about the nature of light would have to be formulated in a considerably more detailed way than had been the case before. Quantification played an increasingly important role in this process. In the wake of the rise of quantification, a thorough mathematisation of the natural philosophical tradition in optics took place. At the same time, mainly through the early work of Malus, the range of the mathematical tradition in optics was extended beyond the eighteenth-century theory of the telescope. Therefore, at the beginning of the nineteenth century, the two traditions in optics came considerably closer. For one thing, one could no longer distinguish the two by simply saying that one was mainly qualitative and the other mainly quantitative.

In the early decades of the nineteenth century, in connection with this apparent convergence of the two traditions in optics, it was questioned to what extent the mathematical tradition of optics in its most general form was really independent of any theory about the nature of light. More specifically, the question was what could be said about the phenomena of light on the basis of the assumption of the existence of light rays only. In the end, again under the influence of Malus, it was found that the eighteenth-century distinction between a mathematical and a natural philosophical tradition in optics could be largely maintained. Just as before 1800, the two traditions continued to go their own way, developing virtually independently of one another. Whereas the natural philosophical tradition developed into a field that was referred to as *physical optics*, the mathematical tradition developed into a field that became known as *geometrical optics*.

Etienne Malus

Although Etienne Malus is far better known for his work on physical optics and his discovery of polarisation, he also contributed to geometrical optics. Indeed, precisely because he contributed to both fields, Malus played a crucial role in the transition from the eighteenth-century mathematical and natural

[14] Cf. Buchwald (n. 2), *The Rise of the Wave Theory of Light*.

philosophical tradition to nineteenth-century geometrical and physical optics.[15] In his long "Traité d'Optique analytique" of 1808,[16] Malus studied the question of what could be said about the phenomena of light on the basis of the assumption of the existence of light rays only. In order to do this, he first considerably extended the range of eighteenth-century catoptrics and dioptrics. Whereas these theories were mainly concerned with the behaviour of rays in the plane and the formation of images, Malus considered two-dimensional systems of rays in ordinary space as well as the systems they gave rise to after reflection and refraction at a surface according to the ordinary sine law of refraction. In doing so, he followed his teacher Gaspard Monge (1746-1818).[17] As John Herivel has argued, the latter may be considered as a precursor of the "analytico-positivistic" approach to physics of the early nineteenth century as opposed to the "mechanico-molecular" approach in the same period. Whereas the latter approach, as advocated by Laplace and his followers, aimed at the reduction of all phenomena to the movement of molecules, the former approach concentrated on a purely mathematical, preferably geometrical, description of the phenomena of physics. In view of the relatively advanced mathematisation of some of its parts, the field of optics clearly constituted a subject that must have seemed amenable to such a mathematical treatment.[18] Relying heavily on the

[15] Etienne Malus, "Traité d'optique," *Journal de l'École Polytechnique*, Cah. 14 (= vol. 7), pp. 1-44, 84-129. A revised version of this paper was published twice. See *Théorie de la double réfraction de la lumière dans les substances cristallisées. Mémoire Couronné par l'Institut (...)* (Paris, 1810), pp. 8-94 and *Mémoires présentés à l'Institut des Sciences (...). Sciences mathématiques et physiques* 2, 1811, pp. 214-302.

[16] There is some confusion about the actual title Malus gave his work. The paper in the *Journal de l'École Polytechnique* does not have a title. Essentially, however, this paper was the same as the one he had presented to the Académie des Sciences as a "Traité d'optique analitique" the year before (see *Procès-verbaux des séances de l'Académie, tenues depuis la fondation de l'Institut jusqu'au mois d'août 1835*, 10 vols. (Paris, 1910-1922), vol. 2, pp. 606-607). As the later versions of his work are called "Théorie (...)," it is appropriate to refer to the first publication of his work as the "Traité d'optique analytique."

[17] Before the revolution, Monge taught mathematics at the Royal School of Military Engineering at La Mézière. After 1789, he taught geometry at the Ecole Centrale and its successor, the Ecole Polytechnique. Malus was Monge's student at the former. For more on Monge, see René Taton, *l'Oeuvre scientifique de Gaspard Monge* (Paris, 1951).

[18] See John Herivel, "Aspects of French Theoretical Physics in the Nineteenth Century," *British Journal for the History of Science* 3, 1966, pp. 109-132, esp. p. 121. It may be worthwhile to remark here that there is no contradiction between the designation "analytico" in Herivel's "analytico-positivistic approach" and the geometrisation advocated by the proponents of this approach. The opposition between geometry and analysis as we now know it only began to be made by the middle of the 19th century. That even then this distinction was not strongly felt is borne out by for instance the fact that until roughly the 1900s, the French term *géométrie* was also used to designate the whole of mathematics. Around 1800, "analysis" referred to a mode of reasoning in mathematics, not to the kind of mathematics that was used. Its opposite was "synthesis". Of course,

new analytical geometry for three-dimensional space that had been formulated by Monge,[19] Malus developed a geometry of rays and applied this theory to his optical systems. In this way, he hoped to find out not only what could be said about the various phenomena of optics, but also what could be said about the tenability of the different theories of light. An important question he discussed, for instance, was whether a normal system, i.e., a system of normals to a surface, will still be normal after it is reflected or refracted. It seems plausible that this question was inspired by the problem of the physical nature of light. Indeed, according to the wave theory of light a wave front perpendicular to the rays always has to exist. Thusfar, it had not been investigated whether this postulate was mathematically tenable. Nowhere, however, did Malus explicitly comment on this.

In the end, Malus was not very clear about what conclusions he could draw from his findings. Already in the first lines of his paper, he has it that "all optical phenomena arise from the different modifications that the rays of a system experience after reflection or refraction."[20] Nowhere in his memoir, however, does he even attempt to explain such phenomena as diffraction or dispersion. Indeed, given his interest in all optical phenomena, he must have realised that a geometry of rays would not be adequate to provide a complete theory of optics.

In this respect, it is highly interesting to see how two years later Malus delineates the range of applicability of his geometry of rays. In 1810, Malus published a long memoir in which he re-worked his geometry of rays in such a way that it could also be applied to the theory of double refraction. In order to do this, he had to introduce various assumptions of a physical nature. His geometry of rays, however, still played an important part. Therefore, by way of introduction, he also included his memoir of 1808, the text of which he left practically unchanged. As he probably felt compelled to distinguish the contents of this memoir from his experimental and physical work on double refraction, he adorned his work on the geometry of rays and its application to optics with the

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this is also how we have to interpret the "analytique" in the full title of Malus' paper.

[19] Before the French Revolution, Monge published a number of seminal papers in spatial geometry. A good impression of the nature of his lessons and his approach to geometry can be gleaned from the notes on separate sheets which in the course of 1795 were published after each of Monge's lecture at the Ecole Polytechnique and which were collected under the title *Feuilles d'Analyse appliquée à la Géométrie* (Paris, 1801). In 1805, a more easily available, reworked and extended version of this collection was published under the title *Application de l'analyse appliquée à la géométrie*. In the course of the 19th century, this book went through as much as five editions, the last in 1850. Because of his thorough analytical reformulation of spatial geometry, he could be considered as one of the founders of 19th-century (spatial) analytical geometry and, by extension, of modern differential geometry.

[20] Malus (n. 15), "Traité," on p. 1.

subtitle "about all phenomena of light that depend on mathematics (*géométrie*)."[21] From the introduction to the memoir on double refraction as a whole, it can be inferred that by these phenomena he meant precisely such phenomena of ordinary optics as can be explained by mathematical laws that do not depend on the kind of optical theory that is being propagated. Thus, in the end Malus had decided to retain the eighteenth-century distinction between the mathematical theory of optics and the theories about the nature of light. At the same time, by extending the eighteenth-century mathematical tradition in optics to his geometry of rays, he pointed the way to a considerable enlargement of the range of applicability of a mathematical theory of optics.

After the publication of his work on double refraction, Malus himself was never to return to his geometry of rays, neither was his work widely followed-up on by others. At the time, although the relevance of Malus' approach seems to have been recognised within the French Academy of Science, probably most considered his work to be too abstract to be of any practical use.[22] Only after the 1830s, when geometrical optics had, largely independently of Malus, developed into a more theoretical field, did his work receive more than passing attention and began the "Traité" to be viewed as a fundamental contribution to geometrical optics. One might wonder, however, how much of this appreciation was purely historical. The actual relevance for the development of geometrical optics after 1830 is very hard to fathom.

The reception of Malus' "Traité"

The publication of Malus' "Traité" certainly did not pass unnoticed. If perhaps its first publication escaped the attention of most of his colleagues, it did attract attention as the first part of his widely-read memoir on double refraction. Still, the impact of Malus' work on geometrical optics was far less than that of his work on physical optics a few years later. Surely, in the 1820s and 1830s a (small) number of papers inspired by the "Traité" did appear. Like Malus' work itself, however, hardly any of these papers directly concentrated on concrete problems in the construction of optical instruments. Instead, they mostly discussed purely mathematical problems arising from Malus' work. Because of the similarity both in spirit and in content of these papers to Malus' "Traité," I will refer to the whole of these papers as the *Malusian tradition*.

Perhaps not surpringly, most of the authors who worked in the Malusian

[21] See Malus (n. 15), *Théorie*, p. 5.

[22] See *Procès-verbaux des séances de l'Académie [des sciences] (...)*, 10 vols. (Paris, 1910-1922), vol. 3, pp. 183-184.

tradition were French and formed part of the Mongian tradition in geometry. Important contributions to the Malusian tradition were made by the French naval officer and mathematician Charles Dupin (1784-1874) and by Joseph-Diez Gergonne (1771-1859), professor of mathematics at Montpellier and the editor of the first mathematical periodical *Annales des mathématiques pures et appliquées*. In connection with the investigation of the so-called caustics, the theory of which is related to that of systems of rays, the role of the Belgian mathematician and later statistician Adolphe Quetelet (1796-1874) has to be stressed.[23] Around 1830, this Malusian tradition reached its culmination in the work of the Irish Astronomer Royal William Rowan Hamilton (1805-1865). In 1828, Hamilton published a long paper entitled "Essay on the Theory of Systems of Rays" in which he developed a completely systematic approach along the lines set out by Malus. In the course of the 1830s, he published three supplements to this "Essay" in which he showed that by admitting the so-called principle of least path, he could even extend his approach to the study of arbitrary media.[24]

All of these investigations in Malusian vein were very mathematical in nature. Usually, apart from the assumption of the existence of light rays and the use of the sine law of refraction, in principle no physical assumptions were used. As Hamilton put it,

> Whether we adopt the Newtonian or the Huygenian, or any other physical theory, for the explanation of the laws that regulate the lines of luminous or visual communication, we may regard these laws themselves, and the properties and relations of these linear paths of light, as an important separate study, and as constituting a separate science, called often *mathematical optics*. This science of the laws and relations of luminous rays, is, however, itself a branch of

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[23] See Atzema (n. 1), *The Structure of Systems of Lines*, esp. chapters 3 and 4. A caustic curve is the envelope of a plane, one-dimensional system of lines after reflection or refraction in another curve, i.e. it is the unique curve to which all lines of the reflected or refracted system are tangent. Similarly, a caustic surface is the envelope of a reflected or refracted two-dimensional system of lines in space.

[24] William Rowan Hamilton, "Essay on a Theory of Systems of Rays," *Transactions of the Royal Irish Academy of Sciences* 15, 1828, pp. 69-174 (= *Collected Mathematical Papers*, vol. 1, pp. 1-106). The three supplements are W.R. Hamilton, "Supplement to an Essay on the Theory of Systems of Rays," *Ibid.* 16 (1), 1830, pp. 1-61 (= *Collected Mathematical Papers*, vol. 1, pp. 107-144); "Second Supplement" (= *Collected Mathematical Papers*, vol. 1, pp. 145-163) and "Third Supplement to an Essay on the Theory of Systems of Rays," *Ibid.* 17 (1), 1837, pp. 1-144 (= *Collected Mathematical Papers*, vol. 1, pp. 163-293). The principle of least path states that light passing through a medium or various media will follow the shortest optical path, i.e. such a path that it will pass through in the shortest time possible. Like the principle of least action, the principal of least path is a minimal principle and the mathematics that goes with it is the variational calculus (as Hamilton used) or its geometrical counterpart, the theory of contact transformations (as used at the end of the 19th century).

another more general science, which may perhaps be called the theory *of Systems of Rays*.[25]

Apparently, Hamilton himself did not see any contradiction between this definition and his own acceptance of the physically inspired principle of least path. Nor did he seem to have cared much about the emphasis on surfaces orthogonal to a system of lines, rather than on the system itself that goes with the use of the principle. In this, however, he was quite exceptional. Although surfaces normal to a system of rays were studied, they were not explicitly considered as a wave surface. In fact, the only context in nineteenth-century optics at large in which wave surfaces were explicitly studied was that of double refraction in bi-axial crystals.[26] Essentially, all investigations in the Malusian style concerned the geometry of lines and not of surfaces. As such they prefigured the rise of line geometry as a field of study in the second half of the nineteenth century.

In the context of geometrical optics proper, by contrast, there was very little serious interest in Malus' approach. When discussing the reception of the "Traité" among those interested in the problems of optics, it is best to distinguish between the situation in France and that elsewhere in Europe. In fact, the reasons for the lukewarm reception of Malus' work in France were completely different from that outside France.

In England, Germany and most other European countries except France, there certainly was a well-developed interest in geometrical optics. In England, in particular, the optical industry was thriving and traditionally some geometrical optics was part of the Cambridge curriculum. The prevailing attitude towards geometrical optics, however, was very practical and not only in England but also in Germany, there was a marked reluctance to study geometrical optics in such a general and abstract way as propagated in the "Traité." In a review of Malus' memoir on double refraction, for instance, Thomas Young (1773-1829) assumed that the part about optical phenomena in general, i.e., the original "Traité," "will probably be thought, by most English readers, unnecessarily intricate." Besides,

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[25] William Rowan Hamilton, "On a General Method of Expressing the Paths of Light, and of the Planets, by the Coefficients of a characteristic Function," *Dublin University Review* 1833, pp. 795-826, on p. 800.

[26] Throughout the first half of the 19th century, mention of a wave surface was usually connected to the so-called *Wave Surface of Fresnel*, i.e. the wave front to a beam of light departing from a point within a bi-axial crystal and first observed by Fresnel. Following Malus, most people viewed this subject as part of physical optics. On this surface, see Buchwald (n. 2), *The Rise of the Wave Theory of Light*, pp. 260-290. For a more mathematical point of view, see Horst Knörrer, "Die Fresnelsche Wellenfläche," in H. Knörrer a.o., *Arithmetik und Geometrie. Vier Vorlesungen* (Basel, 1986), pp. 115-141 (= *Mathematische Miniaturen*, vol. 3).

he was of the opinion that this part did not appear to contain any "material novelty." In Germany a similar attitude prevailed.[27] By and large, outside of France, the view of geometrical optics as a theory of optical instruments that was typical for the eighteenth century remained prevalent. In most countries, geometrical optics was still limited to such investigations as were considered directly relevant to the construction and improvement of optical instruments. To some extent, the range of investigations which were considered relevant had broadened, but it is not very likely that Malus' work was of any direct influence here. As we remarked above, during the eighteenth century, for instance, the study of dioptrics and catoptrics had concentrated on the formation of images and the work of Euler and others on aberrations had received little recognition. In the 1820s and 1830s, by contrast, the systematic studies on aberration by the British astronomers John Herschel (1792-1871) and George Airy (1801-1892) already attracted far more attention.[28]

On the whole, the proponents of the more practical attitude to geometrical optics as could be found in England and Germany had little interest in Malus' very general approach to the field, neither did they care much for the particular mathematical problems this stance involved. According to Hamilton, for instance, the first time he and Airy met, the latter maintained that the existence of a surface normal to a system of lines after reflection or refraction required no proof. Quite likely, Airy considered that the existence of such a surface was *a priori* established by the wave theory of light.[29] In the 1810s, Carl Friedrich Gauss (1777-1855) had given a very rigorous reformulation of eighteenth-century dioptrics, but no direct influence of Malus was visible.[30] In the same decade, when Gauss' student Johann Franz Encke (1791-1865) became interested in geometrical optics and asked Gauss to lecture on it, the latter refused to do so.

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[27] Thomas Young, "On the Mechanism of the Eye," *Philosophical Transactions* 91, 1801, pp. 23-88.

[28] John Herschel, "On the Aberrations of Compound Lenses and Object Glasses," *Philosophical Transactions* 111, 1821, pp. 222-266; George B. Airy, "On the Principles and Construction of the Achromatic Eye-Pieces of Telescopes, and on the Achromatism of Microscopes," *Transactions of the Cambridge Philosophical Society* 2, 1827, pp. 227-252 and "On the Spherical Aberration of the Eye Pieces of Telescopes," *Ibid.* 3, 1833, pp. 1-64. It has to be said, however, that even their work remained relatively unknown. On this and the relation of Airy's and Herschel's papers to Euler's work, see Fellmann (n. 10), "Leonard Eulers Stellung in der Geschichte der Optik."

[29] Hamilton and Airy probably met in the early 1820s. On Hamilton's remark, see Robert Graves, *The Life of Sir William Rowan Hamilton*, 3 vols. (Dublin, 1882-1889), vol. 2, p. 55.

[30] These investigations were to be published in 1851, see n. 39.

Instead, he recommended Euler's work and Abel Bürja's *Anleitung* of 1793![31] A decade after Encke made his request, another student of Gauss and soon-to-be-colleague, Eduard Schmidt (1803-1832) started to work on an extensive textbook on what he called "analytical optics" as opposed to the physical theory of light. However, even though one would say from the size of the book as it was finally published that it must have been Schmidt's ambition to go beyond Bürja and Euler, his work was still firmly rooted in the eighteenth-century tradition and did not show any influence by Malus' work.[32] In the 1820s, yet another pupil of Gauss, August Ferdinand Möbius (1790-1868), also developed an interest in geometrical optics, but the papers that he devoted to the subject were also very much part of the eighteenth-century tradition of dioptrics and catoptrics. Taking the basics of this kind of optics for granted, he concentrated on the simplification of the calculations involved.[33]

Herschel's influential essay "On Light" may be taken as a last illustration of the reception of Malus' "Traité" outside France. The original English edition of this essay came out in 1830. The manuscript, however, had been completed in 1827 and translations of it appeared almost simultaneously with the English publication. In this work, Herschel does not yet explicitly refer to something like geometrical optics, but there is a large section on the reflection and refraction of light rays. Most of its contents, however, goes back to Smith's *Compleat Treatise* of 1738 and Herschel's own investigations. The only reference to Malus is to be found in the list for further reading. The same goes for Schmidt's translation of Herschel's essay into German of 1831. Only in Pierre Verhulst's translation of the work into French, is attention paid to the Malusian tradition in the form of a short paper by Hamilton on his own optical work together with a number of appendices on Gergonne's and Adolphe Quetelet's theory of caustics. Especially where the last appendices are concerned, however, we probably should not interpret their inclusion as an additional indication of an interest in a more

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31 Abel Bürja, *Anleitung zur Optik, Katoptrik und Dioptrik* (Berlin, 1793). See Carl Bruhns, *Johann Franz Encke - Sein Leben und Wirken (bearbeitet nach dem schriftlichen Nachlaß)* (Leipzig, 1869), on p. 31. Abel Bürja (1752-1816) was professor of mathematics at the (French-speaking) "Académie Militaire" at Berlin.

32 See Eduard Schmidt, *Lehrbuch der analytischen Optik*, ed. Carl Wolfgang Benjamin Goldtschmidt (Göttingen, 1834). From 1831 until 1832, Schmidt had an extraordinary professorship in mathematics at Göttingen. In 1832, he obtained an ordinary professorship in mathematics, astronomy and physics at Tübingen. Within a few months from his arrival in the latter town, however, he died.

33 Ferdinand Möbius, "Kurze Darstellung der Haupteigenschaften eines Systems von Linsengläsern," *Journal für die reine und angewandte Mathematik* 5, 1829, pp. 113-132 (= *Gesammelte Werke*, vol. 4, pp. 477-501) and "Beiträge zu der Lehre von den Kettenbrüchen nebst einem Anhange dioptrischen Inhalts," *Ibid.* 6, 1830, pp. 215-243 (= *Gesammelte Werke*, vol. 4, pp. 503-539).

abstract approach to geometrical optics. In fact, it was Quetelet himself who was responsible for the appendices of Verhulst's translation.[34]

In France, the situation was rather different from that in the rest of Europe. To a large extent, the reticence concerning Malus' geometry of rays which prevailed outside France and its cultural satellites was not found in France itself. Indeed, as explained above, whereas in most other countries the straightforward and practical style that was typical of the eighteenth century survived well into the nineteenth century, in France, especially since the Revolution of 1789, a high degree of mathematisation had become *de rigueur*. In the case of optics, this meant that there was virtually no interest in geometrical optics, English or German style. I have not been able to find any substantial, original publication on dioptrics and catoptrics in the English or German style for the period 1810-1830. Before the 1830s, Malusian optics was the only kind of geometrical optics there was. Even this kind of geometrical optics, however, was only of marginal importance in the context of French science. In fact, those who wrote on geometrical optics worked in the periphery of French science and had little influence. It has to be remembered, for instance, that Dupin did not have a central position in French mathematics; Gergonne did not have a relation to the central circles in French science at all. The crucial point is that the hard-core of French science simply was not interested in geometrical optics in any form whatsoever. During the first three decades of the nineteenth century, the attention of the French scientific elite focused on physical optics. Essentially, optics in France during this period were the investigations of Arago, Fresnel and others.

Only after the 1820s, were there signs of increasing interest in geometrical optics in the more central circles of French science. The interest, however, above all concerned such theory as would be of direct use to the construction of telescopes and other rather practical matters. There was very little interest in the abstract Malusian theory. An interesting case in point here is the work of Jean-Baptiste Biot. As we saw previously, at the beginning of the century he was one of the main proponents of physical optics in France. Likewise, he became one of the champions of a theoretical approach to the construction of optical

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[34] John Herschel, "On Light," in E. Smedley, J. Rose and H.J. Rose (ed.), *Encyclopaedia Metropolitana (...)*, 26 vols. (London, 1817-1845), vol. 2 (1830), pp. 341-586. As for the translations, see: *Vom Licht*, trad. Eduard Schmidt (Stuttgart & Tübingen, 1831); *De la Lumière*, 2 vols., trad. Pierre Verhulst (Bruxelles, 1829-1831). On Quetelet's caustics, see Atzema (n. 1), *The Structure of Systems of Lines*, ch. 3. Hamilton's contribution, "Sur l'emploi d'une formule générale propre à résoudre les differentes questions d'optique," in vol. 2 of *De la Lumière*, on pp. 456-468, was translated from the original English by Quetelet. This abstract was not included in Hamilton's *Mathematical Papers*.

instruments in the 1840s. This is not to say that before this period he had had no interest in the subject at all. In fact, already at the beginning of this career, he had had contacts with a number of instrument makers, in particular with Robert-Aiglé Cauchoix (1776-?).[35] The scientific climate at the time, however, was such that he could not propose that serious attention be paid to the formulation of a "modern" theory behind the construction of optical instruments.[36] In the early 1840s, this situation had fundamentally changed and Biot had no diffulties getting his very technical notes on geometrical optics and its application to the construction of optical instruments published in the *Comptes-rendus*.

Thus, the appreciation of geometrical optics in France gradually converged to that in the rest of Europe. As in the case of England and Germany, the practice of the construction of optical instruments became the main motivation behind geometrical optics. After the 1830s, this tendency was very outspoken in the whole of Europe and geometrical optics was more and more appropriated by two rather practically-minded professional groups within the scientific community that were intent on the formulation of a theory that might serve their practical goals. First of all, there were the astronomers, who tended to view geometrical optics as a theory that served as a mere back-ground to the construction of optical instruments. A second group was formed by those who were interested in the functioning of the eye. Among the members of both these groups, the very general Malusian approach to geometrical optics was not considered to be of direct relevance. Gradually, however, a more abstract approach to geometrical optics very similar to the one advocated by Malus began to prevail as a result of the continuing 'theoretisation' of the field.

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[35] Cauchoix constructed a number of optical instruments for Biot. Also, Biot supported Cauchoix with the introduction of Wollaston's so-called periscopic glasses in France. See Biot, "Notice sur un nouveau genre de Bésicles, inventé par le célèbre physicien M. Wollaston," *Moniteur Universel*, 21-09-1813 (p. 1044).

[36] An interesting illustration of this might be a report on the production of flint glass that Biot wrote for the *Académie des Sciences* in 1811. In this report, he refers several times to an appendix on some rather technical calculations concerning the spherical aberration of an optical instrument. In none of the published versions of the report, however, is this appendix to be found. It seems, he left out these calculations at the last moment. An explanation for this might be that nobody would have been interested. See Aimé-Gabriel d'Artigues, *Sur l'art de fabriquer du flintglass pour l'optique, suivi d'un Rapport fait à la Classe des Sciences Physiques et Mathématiques de l'Institut sur les resultats de cette fabrication* (Paris, 1811) (Rapport de Biot), also published elsewhere.

The astronomical community and the rise of technical optics

In the above, we already saw how astronomers like Herschel, Airy and Encke showed an interest in geometrical optics as a means to improve the construction of optical instruments. This view on geometrical optics was clearly a continuation of the view that most people held with regard to the main goals of dioptrics. A similar attitude towards geometrical optics continued to persist throughout the nineteenth century, especially among astronomers.

In the early 1840s, this interest on the part of the astronomical community received a fresh impetus through the publications of Friedrich Wilhelm Bessel (1784-1846) and Gauss. In 1841, Bessel published a short note on the theory of the formation of images. About the same time, Gauss finally decided to publish his optical investigations of twenty years earlier; in these investigations he followed a line of approach similar to that of Bessel.[37] Both Gauss' and Bessel's work accorded excellently with the interest of the astronomical community in a more sophisticated version of the eighteenth-century mathematical optical theory. With some delay, their papers – especially Gauss' "Dioptrische Untersuchungen" of 1841 – gave rise to a spate of publications in which their theory was further developed. In the modern literature, this theory is usually referred to as Gaussian or paraxial optics, since it is only concerned with rays very near to a central axis. In addition to initiating the study of paraxial optics, the investigations of Gauss and Bessel led to renewed interest in the theory of aberrations. In 1843, an abstract of Gauss' theory was published in Taylor's *Scientific Memoirs*.[38] In 1851, Auguste Bravais (1811-1863) published a French translation of the "Dioptrische Untersuchungen," together with a short note.[39] A decade later, in 1856, this translation and Bravais' note to it were published a second time, together with an extract of Bravais' lessons at the Ecole polytechnique, in which he followed up on Gauss' work.[40] In the course of the same

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[37] Friedrich Wilhelm Bessel, "Ueber die Grundformeln der Dioptrik," *Astronomische Nachrichten* 18, 1841, pp. 97-108; Carl Friedrich Gauss, "Dioptrische Untersuchungen," *Abhandlungen der königlichen Gesellschaft der Wissenschaften zu Göttingen* 1, 1841, pp. 1-34 (= *Gesammelte Werke*, vol. 5, pp. 243-276).

[38] William Miller, "Abstract of some of the principal Propositions of Gauss' "Dioptric Researches"," *Scientific Memoirs* 3, 1843, Part 9, pp. 490-498.

[39] Carl Friedrich Gauss, "Recherches de Dioptrique," *Annales de chimie* 33, 1851, pp. 259-294, trad. Auguste Bravais; Auguste Bravais, "Note de dioptrique," *Annales de chimie* 33, 1851, pp. 494-501.

[40] Carl Friedrich Gauss, "Recherches de Dioptrique," *Journal des mathématiques pures et appliquées* (2) 1, 1856, pp. 9-43; Auguste Bravais, "Note de dioptrique," *Ibid.*, pp. 44-51 and "Résumé succint des formules de Gauss sur la théorie des lunettes, et leur application à la démonstration des propriétés de l'anneau oculaire. Note tirée des leçons de M. Bravais à l'Ecole

decade, the theory of the principal aberrations in optical instruments was brought to perfection. The contribution made to this theory by the Munich professor of mathematics Ludwig Seidel (1821-1896) is best known, but similar work was done by Ottaviano Mossotti (1791-1863) in Milan. A decade earlier, the nestor of French science Jean-Baptiste Biot, whom I mentioned earlier, and the Vienna professor of mathematics Joseph Petzval (1807-1891) had come close to an equivalent theory.[41] With the exception of Petzval, all of those just mentioned were astronomers (Gauss, Bessel, Biot, Mossotti) or had close ties with the astronomical communities of their countries (Seidel had studied astronomy with Encke, Bravais had taught astronomy at the Ecole polytechnique). In the course of the second half of the nineteenth century, this concern about dioptrics and its application to the construction of instruments continued to be typical of the astronomical community in most European countries.

Of course, the interest in geometrical optics of the astronomical community is not really surprising. After all, since Kepler's times, progress in observational astronomy was directly linked to the improvement of the performance of the telescopes the astronomers had at their disposal. Since the beginning of the nineteenth century, however, another factor was becoming increasingly important as well. Whereas before 1800, the astronomers may have had an interest in the improvement of telescopes, their influence on the actual practice of construction was minimal. In general, the instrumentmakers were very practical artisans with a marked aversion for the theoretical approach to the construction of instruments that an astronomer was likely to follow. By the end of the eighteenth century, this situation was gradually changing, especially in Germany. At the beginning of the nineteenth century, for instance, the self-made opticians Joseph Fraunhofer (1787-1826) and Johann Georg Repsold (1770-1830), both definitely advocated a more theoretical approach to the development and

polytechnique," *Ibid.*, pp. 51-59.

41 Jean-Baptiste Biot, *Traité élémentaire d'astronomie physique*, third ed., 5 vols. (Paris, 1841-1857), vol. 2, 1843, pp. 1-63; "Sur quelques points rélatifs à l'astronomie et aux instruments d'optique," *Comptes-Rendus Hebdomadaires de l'Académie des Sciences* 12, 1841, pp. 269-276; "Sur un Mémoire de M. Gauss, rélatif à l'optique analytique," *Ibid.*, pp. 407-413; "Sur la formation directe des coefficients généraux des systèmes optiques," *Ibid.*, pp. 519-523 (with comments by Libri); "Sur les lunettes achromatiques à oculaires multiples," *Ibid.* 13, 1841, pp. 1039-1046 and "Note sur quelques points d'optique mathématique,' *Ibid.* 19, 1844, pp. 495-500; Joseph Petzval, *Bericht über die Ergebnisse einiger dioptrischen Untersuchungen* (Pesth, 1843); Ludwig Seidel, "Zur Theorie der Fernrohr-Objective," *Astronomische Nachrichten* 35, 1852, pp. 301-316; "Zur Dioptrik," *ibid.* 37, 1854, pp. 105-120 and "Zur Dioptrik. Ueber die Entwicklung der Glieder 3ter Ordnung, welche den Weg eines ausserhalb der Ebene der Axe gelegenen Lichtstrahles durch ein System brechender Medien bestimmen," *ibid.* 43, 1855, pp. 289-332; Ottaviano Mossotti, "Nuova teoria degli strumenti ottici," *Annali della Università Toscana. Parte Secunda: Scienze Cosmologiche* 4, 1855, pp. 39-165 and 5, 1861, pp. 7-140.

production of optical instruments.[42] Where the former tried to solve his theoretical problems mostly by himself, the latter simply turned to Gauss, who by then was mainly known as an astronomer.[43] This choice of an astronomer to consult with is of course not really surprising and there were indeed many other opticians who turned to the astronomers for their theoretical problems. Clearly, the astronomers had the knowledge and interest to solve most problems in the theory of optics. Besides, they were the principal clients of the opticians. In this way, the astronomer gradually obtained the position of *advisor* to the optical industry.

In the beginning, this interaction between the astronomical community and the opticians was fruitful for both parties. In exchange for the solution of theoretical problems, the astronomers obtained better telescopes. In the course of the nineteenth century, however, the production of optical instruments that were not directly relevant to astronomy became increasingly important. Especially after the invention of photography in the late 1830s, the problems the opticians wanted to have solved became of ever less importance to the astronomical community. The work of Petzval, for instance, concerned the improvement of photographical lenses. The production of optical instruments was gaining an impetus of its own. As a consequence, those who worked as advisors on matters theoretical were more and more forced into a subservient position.

With the rise of large-scale optical industry in the early 1870s, the cooperation between the astronomical community and the community of opticians that had been based on the often informal exchange of theoretical knowledge for instruments, was no longer satisfactory. Gradually, the theory of instruments developed into a discipline of its own with its own specialists. Already in the 1830s, the astronomer Carl Steinheil (1801-1870) had turned away from astronomy to concentrate on the construction of optical instruments on the basis of his knowledge of geometrical optics. In the course of the century, others followed suit. An important stage in this process was reached when in the late 1870s Carl Zeiss hired Ernst Abbe (1840-1905), a physician and former assistent to the Göttingen astronomer Wilhelm Klinckerfuß, to provide scientific assistance for the production of microscopes by his company.[44] By then, *technical* optics, as the theory began to be referred to, had definitely acquired a

[42] On this, see Jackson (n. 5), "Die britische Antwort," on pp. 7-12 and the literature mentioned there.

[43] See P. Riebesell, "Briefwechsel zwischen C.F. Gauss und J.G. Repsold," *Mittheilungen der Mathematischen Gesellschaft in Hamburg* 6, 1928, pp. 398-431.

[44] See Joachim Wittig, *Ernst Abbe* (Leipzig, 1989) (= *Biographien hervorragender Naturwissenschaftler, Techniker und Mediziner*, vol. 94).

place of its own in the spectrum of technical sciences that was becoming increasingly visible towards the end of the nineteenth century. The various handbooks on technical optics that were written under the direction of Abbe and his successors helped a lot to provide firm foundations for this field.[45] In 1902, at the Technische Hochschule in Berlin, the mathematician Alexander Gleichen (1862-1923) became the first *Privatdozent* in technical optics in Germany.[46] In England and France, the teaching of technical optics was institutionalised during or shortly after the First World War.[47]

Physiological optics and the rise of the dioptrics of the eye

Technical optics was not the only kind of geometrical optics that had come into being by the end of the nineteenth century. In the course of the second half of the nineteenth century, the study of the optical properties of the eye was acquiring a status of its own as well.

During the first decades of the nineteenth century, the study of the eye in general and its optical properties in particular had not been pursued systematically. Traditionally, the study of the eye was an important part of physiology. In the first half of the century, however, physiology underwent a profound change. Intrusive experiments like vivisection began to play an important role, especially in France. By laying bare the organs of living animals and observing the processes that took place, physiologists like François Magendie (1783-1855) obtained important new insights into the functioning of organs and their interdependence. Because of its delicacy, the eye was hardly suited for this rough kind of experimentation. Characteristically, the small number of publications on the eye which did appear were published in Germany. In that country, a long tradition of non-intrusive observation prevailed until well

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[45] Cf. Siegfried Czapski, *Theorie der optischen Instrumenten nach Abbe* (Breslau, 1893); Moritz von Rohr (ed.), *Die Bilderzeugung in optischen Instrumenten vom Standpunkte der geometrischen Optik. Bearbeitet von den wissenschaftlichen Mitarbeitern an der optischen Werkstätte von Carl Zeiss* (Berlin, 1904).

[46] Alexander Gleichen could count as the first academic specialist in geometrical optics. Already his inaugural dissertation of 1888, written under the direction of Gustav Karsten at Kiel, was on a subject in geometrical optics. Gleichen would continue to teach geometrical optics at Berlin until 1919, when he started to work full-time for the optical firm Görz. He also wrote a *Lehrbuch der geometrischen Optik* (Berlin, 1902), which is more a textbook on technical optics, a *Einführung in die medizinische Optik* (Berlin, 1904), as well as numerous other works on matters optical.

[47] Cf. Joachim Wittig (n. 5), "Zur Disziplingenese der technischen Optik"; Mari Wiliams, "Crisis or Complacency? The Precision Instrument Industry in Britain and France, 1900-1920," in Chr. Blondel e.a. (ed.), *Studies in the History of Scientific Instruments* (London, 1989), pp. 273-281.

into the nineteenth century. In the context of his ophthalmological investigations in the course of the 1820s, Evangelista Purkinje (1787-1869) at Breslau paid considerable attention to the physiology and the optical properties of the normal eye. Also in the 1820s, the Berlin physiologist Johannes Müller (1801-1858) studied various properties of the eye.

By the late 1830s, this relative neglect of the study of the eye took another turn. In 1836 and 1838, for instance, the Dorpat professor of physiology Alfred Wilhelm Volkmann (1801-1877) published two papers on experiments that he had conducted on eyes that he had taken from albino rabbits.[48] Because of the lack of pigment in such eyes, the images of a light source on the retina could be seen from the outside, without opening up the eye ball. This made it possible to study these images in detail. Typical for this kind of research was the way Volkmann tried to quantify his investigations. Whereas most investigations in physiology were largely qualitative in nature, the approach followed by Volkmann and others was more quantitative and was directly inspired by that of the new physics.[49] In the course of the 1840s, this physics-oriented (or physicalist) approach to the study of the eye was extended to the whole of physiology and led to the formation to the famous 1847-Gruppe around Emil du Bois-Reymond (1818-1896), Ernst Brücke (1819-1892) and Hermann von Helmholtz (1821-1894). The central tenet of this very influential group of physiologists was that there was no fundamental difference in the way organic and inorganic matter should be investigated. In other words, physiology should be viewed as a branch of physics. The quantitative study of the eye of the late 1830s probably played a role in the articulation of this physicalist approach. As far as I know, this matter has never been investigated in detail.[50]

In the light of this quantitative approach to the study of the eye, it becomes understandable why Gauss' "Dioptrische Untersuchungen" and Bessel's work aroused a lot of interest. Both Gauss' and Bessel's work provided an easy means to assign a number of quantitative characterics to an optical system. Assuming

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[48] Alfred Wilhelm Volkmann, "Untersuchungen über den Stand des Netzhautbildchens," *Annalen der Physik und Chemie* 37, 1836, pp. 342-352 and "Theorie zur Berechnung der von mir gemessenen Zerstreuungskreise des Lichts, bei fehlerhafter Accomodation des Auges," *Ibid.* 45, 1838, pp. 193-226.

[49] For some of the others who wrote in this vein, see the references in Hermann Helmholtz, *Handbuch der physiologischen Optik*, second ed., ed. A. König (Hamburg & Leipzig, 1896), pp. 1062-1063.

[50] Of course, philosophical convictions played an equally important role here. On this, see Paul Cranefield, "The Organic Physics of 1847 and the Biophysics of Today," *Journal of the History of Medicine* 12, 1957, 407-423; Friedrich Harnack, "Emil du Bois-Reymond und die Grenzen der mechanistischen Weltauffassung," in *Festschrift zur 150-Jahr-Feier der Humboldt-Universität Berlin*, 2 vols. (Berlin, 1960), vol. 1, pp. 229-251.

that their approach could be extended to the system formed by the constituent parts of the eye, one would be able to assign similar characteristics to any eye. Of course, the thought that the eye might be considered as a sophisticated optical instrument was not new. Essentially, the idea can already be found in Kepler's work of the beginning the seventeenth century. To some extent, the study of the eye was even considered part of the pre-nineteenth-century tradition in optics.[51] Until Gauss' and Bessel's investigations, however, it had been difficult to give more than a rough indication of how the eye functioned as an optical instrument. An important reason was that eighteenth-century dioptrics worked with lenses placed in one and the same medium. Since the formation of the image on the retina takes place in a different medium from the air outside the eye, classical dioptrics could not be used for the eye. In contrast to their predecessors, Gauss and Bessel did take into account the possibility that the medium before a lens was different from that behind the lens. As they showed, this extension of the theory did not make it conceptually more complicated. Only the number of calcultions required slightly increased.

Already in 1841, the year of the publication of Bessel's paper, the Königsberg physicist Ludwig Moser (1805-1880) applied Bessel's theory of dioptrics to the study of the eye. A few years later, in 1845, and then more extensively in 1854, the Göttingen physicist Bernard Listing (1808-1877) used Gauss' theory to the same end.[52] Whereas Moser had followed a rather practical and *ad hoc* approach, Listing assumed a point of view that was as mathematical and formal as possible. Instead of straightforwardly computing all kinds of data, he started with the introduction of what he called the schematic eye (*das schematische Auge*). By schematising very carefully all parts of the eye and assigning indices of refraction to them, he ended up with an optical system that could serve as a model of the eye. He now had enough data to compute everything he wanted to know about the eye. In a relatively short time, Listing's

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[51] Cf. the *Encyclopédie*, where the mathematical tradition in optics is defined as "une partie des mathématiques mixtes, où l'on explique de quelle manière la vision se fait, où l'on traite de la vûe en général, où l'on donne les raisons des differentes modifications ou altérations des rayons dans leur passage au-travers l'oeil, & où l'on enseigne pourquoi les objets paroissent quelquefois plus grands, quelquefois plus petits, quelquefois plus distincts, quelquefois plus confus, quelquefois plus proches, quelquefois plus éloignés." See D. Diderot and J. d'Alembert (ed.), *Encyclopédie ou dictionnaire raisonné des sciences, des arts et des métiers*, 35 vols. (Paris, 1751-1780; reprint 1966-1967), vol. 11, pp. 518 (s.v. Optique).

[52] Ludwig Moser, "Ueber das Auge," in Heinrich Wilhelm Dove (ed.), *Repertorium der Physik. Enthaltend eine vollständige Zusammenstellung der neueren Fortschritte dieser Wissenschaft*, 8 vols. (Berlin, 1837-1849), vol. 5, 1844, pp. 337-412; Johannes Benedict Listing, *Studien zur physiologischen Optik* (Göttingen, 1845) (= Ostwald's *Klassiker der Wissenschaft*, vol. 147, 1905); "Mathematische Discussion des Ganges der Lichtstrahlen im Auge," in Rudolph Wagner (ed.), *Handwörterbuch der Physiologie*, 4 vols. (Berlin, 1845-1853), vol. 4, pp. 451-504.

schematic eye together with the underlying optical theory began to be considered as a crucial instrument in the search for the quantification of the functioning of the eye. Due to the invention of more refined experimentation techniques in the course of the 1840s and 1850s, the more physiological aspects of the eye were beginning to be studied as well. However, much of the research in physiological optics, as the physiological study of the eye was beginning to be called, in one way or another centred around the concept of the schematic eye. When in 1856 Hermann Helmholtz (1821-1894) published the first instalment of his *Handbuch der physiologischen Optik*, he duly acknowledged the central role of Listing's concept. In this detailed survey of all physiological investigations of the eye until 1856, the schematic eye was among the first topics to be discussed.

In the course of the last four decades of the nineteenth century, the study of the various problems concerning the schematic eye and its relation to the real eye began to form an important subfield of physiological optics. In this subfield, apart from the schematic eye itself, problems like the determination of images for an optical system in general (as in classical dioptrics) were studied as well. By and large, the lines set out by Gauss and Bessel were followed. Because of the limited background in mathematics of most who worked in physiological optics, an important aim of these studies was further to simplify Gauss' calculations. Many now-classical works in physiological optics, such as Helmholtz' *Handbuch* and Frans Donders' *On the Anomalies of the eye*, contained a large section on optical systems.[53] Usually, the whole of this subfield was referred to as the dioptrics of the eye. According to Stephen Turner, it can be deduced from the references to the second edition of Helmholtz' *Handbuch* of 1896 that about twenty per cent of all publications in physiological optics over the second half of the nineteenth century was devoted to the dioptrics of the eye. Over the period 1890-1894, this percentage had even risen to about 23. After the study of the anatomy of the eye and that of the purely physiological aspects of the eye, this made the dioptrics of the eye the second-most studied problem-complex in the field during this period.[54]

By the 1890s, there were various physiologists, physicists and others who had made the dioptrics of the eye their academic specialisation. Ludwig Matthiessen (1830-1906), professor of physics at the University of Rostock, devoted the lion's share of his publications to the comparative study of the dioptrics of the eyes of

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[53] Heinrich Helmholtz, *Handbuch der physiologischen Optik* (Leipzig, 1856-1867), pp. 35-90; Frans Donders, *On the Anomalies of the Accomodation and Refraction of the Eye. With a Preliminary Essay on Physiological Dioptrics*, transl. from the manuscript (in Dutch) by W.D. Moore (London, 1864).

[54] Richard Stephen Turner, "Paradigms and Productivity: The Case of Physiological Optics," *Social Studies of Science* 17, 1987, pp. 35-68, on p. 44.

vertebrates. In Paris, Marius Tscherning (1854-1934), the director of the ophthalmological clinic of the Sorbonne, was also keenly interested in the mathematical study of the eye. The most important proponent of this new specialisation was the Swedish ophthalmologist and physiologist Allvar Gullstrand (1861-1930). From his doctoral thesis of 1890 onwards until his final publications in the 1920s, he studied almost all aspects of the functioning of the eye that were amenable to mathematical treatment.[55]

In the last decade of the nineteenth century, the dioptrics of the eye was clearly developing its own dynamics, even if it remained confined within the boundaries of physiological optics. As part of this broader field, it tended to attract more attention than the other parts and increasingly, physiological optics became almost synonymous with the mathematical study of the eye. In the third edition of Helmholtz' *Handbuch der physiologischen Optik* of 1909, for instance, the part on the dioptrics of the eye received far more extensive annotations than any other part of the book. No doubt, the fact that in 1911 Gullstrand received the Nobel prize for medicine and physiology for his work on the dioptrics of the eye has to be viewed as a more or less official recognition of the high status that the field had obtained.

Thus, by the end of the nineteenth century, geometrical optics had evolved into at least two distinct, rather different fields. In both these fields, the very general approach to geometrical optics which Malus had advocated did not play an important role. This is not to say, however, that a more abstract and general approach to geometrical optics had disappeared completely. Both in technical and physiological optics, the problematics of these fields gave rise to general questions that were not of direct relevance for the construction of instruments or for the study of the eye. The approach that was followed to solve these problems sometimes came very close to a geometrical optics for its own sake. Less in content than in spirit, the way these problems were solved was very reminiscent of Malus' work. The history of the search for a description of the so-called infinitely thin pencil provides just one example. For any one ray of a system, it was assumed one could define an object that was formed by all the rays of the system 'infinitely near' to this ray, i.e., immediately surrounding it. This object was called the infinitely thin pencil. In my thesis, I explain how this concept of the infinitely thin pencil arose in the context of the explanation of the

[55] On Matthiessen, see Reinhard Mahnke, "Ludwig Matthiessen – Erster ordentlicher Professor der Physik an der Universität Rostock," *Beiträge zur Geschichte der Universität Rostock* 17, 1991, pp. 19-33. On Gullstrand, there is little literature. A rather loquacious obituary is Wilhelm Oseen, "Allvar Gullstrand," *Kungliga Svenska Vetenskapsakademiens Årsbok* 35, 1937, pp. 249-273. As to the biographical facts of Gullstrand's life, see also Charles Snyder, *Our Ophthalmological Heritage* (Boston, 1967), pp. 149-152. There seems to be no study on Tscherning.

phenomena arising from astigmatism of the eye, one of the first successful theories of physiological optics. At the same time, the search for an accurate description of such a pencil was related to yet another problem in physiological optics. This problem concerned the fact that the human eye has an angle of vision that is far larger than that of any man-made optical instrument. In the late nineteenth century, the infinitely thin pencil was used to explain this property.[56]

Two other examples concerned the problem of the determination of the paths of light rays in optically inhomogeneous media. In the first case, this problem occurred in the context of the path of light through the crystalline lens of the eye. Already at the beginning of the nineteenth century, it had been found that this lens consists of material of which the index of refraction increases towards its centre. This meant that the ordinary laws of geometrical optics did not apply. A similar problem had to be dealt with in observational astronomy. In this case, it was the problem of atmospheric refraction that gave rise to the study of an inhomogeneous medium. The problem concerned the exact location of a radiant object such as a star. Every light ray from a star which is received on earth will be deflected on its passage through the atmosphere. Therefore, a correction has to be applied to find out the direction from which the ray really came. For such a correction, one has to take into account the fact that the atmosphere is layered and that its temperature increases the closer one gets to the surface of the earth. This means that, just as the crystalline lens, the atmosphere has to be considered as optically inhomogenous. Both in the case of the crystalline lens and of the terrestrial atmosphere, attempts were made to solve this problem of the determination of the paths of light by considering an optically inhomogeneous medium as an optical system consisting of an infinite number of lenses. Investigations along these lines were pursued by, for instance, the Berlin mathematician Eduard Kummer (in connection with atmospheric refraction) and Matthiessen (in connection with the crystalline lens).[57] However, before the 1890s, no decisive results seem to have been achieved by this approach.

The persistence of the Malusian tradition

The two traditions in geometrical optics which I have just sketched formed the bulk of nineteenth-century geometrical optics. Apart from these traditions,

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[56] On all this, see Atzema (n. 1), *The Structure of Systems of Lines*, chapter 7.

[57] Ernst Eduard Kummer, "Ueber atmosphärische Strahlenbrechung," *Journal für die reine und angewandte Mathematik* 61, 1863, pp. 263-275 (= *Collected Papers*, vol. 2, pp. 337-349); Ludwig Matthiessen, *Grundriß der Dioptrik geschichteter Linsensysteme* (Leipzig, 1877).

however, there was also a minor third one. In fact, such abstract and general problems in geometrical optics as sketched above did not only arise as a mere spin-off of such practical problems as discussed above. Throughout the century, the general (line-geometrical) approach to optics established by Malus continued to be dealt with as a subject in its own right. After the 1830s, a very small trickle of publications on Malusian optics continued to appear, especially in France and countries that were much influenced by French science. Between 1820 and 1840, for instance, the Finnish mathematician Nathanaël af Schultén (1796-1860) devoted quite a number of publications to Malusian optics. Schultén had a student, Henrik Borenius (1802-1894), who devoted most of his publications in mathematics to Malusian optics and the study of systems of lines.[58] In France itself, the work of Charles Sturm (1803-1855) on the relations between the caustic surfaces of a system of lines before and after reflection or refraction should be mentioned.[59] In addition, particular themes in Malusian optics were taken up by the mathematical community. Joseph Bertrand (1822-1900), for instance, showed that the sine law of refraction is the only law for which Malus' theorem concerning the retention of normality after refraction will hold for all normal systems and all refracting surfaces.[60] Similar elaborations on Malus' theorem can be found in the works of Eugenio Beltrami (1835-1900), Alfred Enneper (1830-1885), Sophus Lie (1842-1899) and other mathematicians.[61]

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[58] Cf. Nathanaël af Schultén, "Recherche générale sur la quantité de lumière directe ou indirecte envoyée dans l'oeil par des objets lumineux," *Mémoires présentés à l'Académie Imperiale de Saint Petersbourg par divers Savants* 1, 1831, pp. 39-51; "Note sur les faisceaux infiniment menus, répandus dans l'espace suivant une loi analytique donnée," *Ibid.* 4, 1845, pp. 203-214; and "Note ulterieure sur les faisceaux infiniment menus," *Ibid.*, pp. 215-225; Henrik Borenius, *In theoriam luminis reflexi disquisitio* (Inaug.-Diss. Helsingfors, 1834); "Determinatio superficiei intersectione continua omnium generis dati linearum prodeuntis," *Acta Societatis Scientiarum Fennicae* 1, 1840, pp. 381-385 and "Determinatio superficiei, omnes generis dati lineas dato sub angulo intersecantis," *Mémoires présentés à l'Académie Imperiale de Saint Petersbourg par divers Savants* 4, 1845, pp. 255-272. On Schultén and Borenius, see Gustav Elfving, *The History of Mathematics in Finland, 1828-1918* (Helsinki, 1981) (= O. Ahlbäck e.a. (ed.), *The History of Learning and Science in Finland, 1828-1918*, vol. 17, pp. 20 seq.

[59] Charles Sturm, "Mémoire sur l'optique," *Journal des mathématiques pures et appliquées* (1) 3, 1838, pp. 357-384 and "Mémoire sur la théorie de la vision," *Comptes-rendus hebdomadaires des séances de l'Académie des Sciences* 20, 1845, pp. 554-560, 761-767, 1238-1242.

[60] Joseph Bertrand, "Mémoire sur la théorie des surfaces," *Journal des mathématiques pures et appliquées* (1) 7, 1844, pp. 133-154. Twenty years later, he devoted a complete chapter of his analysis textbook to the subject, see J. Bertrand, "Etude des normales à une surface," in *Traité de calcul différentiel et de calcul intégral*, 2 vols. (Paris, 1864), vol. 2, pp. 672-691 (= ch. 6).

[61] Cf. Eugenio Beltrami, "Ricerche di analisi applicata alla geometria," *Giornale di matematiche pure ed applicate* 2, 1864, pp. 267-282; Alfred Enneper, "Das Theorema von Malus," *Zeitschrift für Mathematik und Physik* 8, 1863, pp. 61-66; Sophus Lie, "Die infinitesimalen Berührungstransformationen der Optik," *Berichte über die Verhandlungen der Königlich Sächsischen Akademie der Wissenschaften zu Leipzig. Mathematisch-physische Klasse* 47, 1896, pp. 131-133 (=

In the last few decades of the nineteenth century, interest in Malusian optics was even growing again, particularly in the textbooks on general optics. In 1869, the posthumous *Leçons sur l'Optique Physique* by Marcel Verdet (1824-1866) on the general theory of light, for instance, began with a summary of Malus' geometry of light rays. Two decades later, Verdet's pupil Eleuthère Mascart (1837-1908) gave an even more detailed discussion of Malusian optics in his own textbook on optics. In England, Robert Heath (1858-1931) paid considerable attention to some aspects of Malusian optics in his *Geometrical Optics* of 1888, although it cannot be said that he embraced the Malusian approach whole-heartedly.[62]

As far as original investigations in geometrical optics were concerned, another student of Verdet, Alfred Lévistal (1838-1874), has to be mentioned. In the early 1870s, after graduating on a topic in geometrical optics in 1866, he devoted two of his last publications to the field as well. A decade later the French army officer and teacher at the Ecole polytechnique Amadée Mannheim (1831-1906) also published on Malusian optics. Meanwhile, James Maxwell (1831-1879) in England had begun to apply Hamilton's theory of geometrical optics to the infinitely thin pencil. Around 1890, his work was followed up by Joseph Larmor (1857-1942).[63]

All in all, by the end of the nineteenth century, the Malusian approach to optics was becoming more visible and acceptable than it had ever been. In fact, whereas at the time of Malus an abstract and mathematically advanced approached was considered undesirable by almost all those interested in geometrical optics, the situation was much more differentiated by the end of the century. By then, there was a large group of scientists with an interest in optics who considered that a more theoretical and abstract approach to geometrical

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Gesammelte Abhandlungen, vol. 6, pp. 615-618). For some of the other papers, see Atzema (n. 1), *The Structure of Systems of Lines*, ch. 4.

[62] Cf. Eleuthère Mascart, *Traité d'Optique*, 3 vols. (Paris, 1889-1894), vol. 1, pp. 61-156 (= ch. 2: "Systèmes optiques"); Robert Heath, *Geometrical Optics* (London, 1888). An authorised translation by R. Kanthack appeared in 1894; see *Lehrbuch der geometrischen Optik* (Berlin, 1894). In this translation, Malusian optics receives even more attention.

[63] Alfred Lévistal, "Etudes d'optique géometrique," *Journal de physique théorique et appliquée* 1, 1872, pp. 209-216, 247-250 and "Généralisation du théorème de Gergonne," *Ibid.* 2, pp. 207-210; Victor-Amadée Mannheim, "Mémoire d'optique géométrique," *Atti della Reale Accademia dei Lincei. Memorie* 1, 1885, pp. 520-546 (= *Journal des mathématiques pures et appliquées* (4) 2, 1886, pp. 5-48); James Clerk Maxwell, "On the Focal Lines of a Refracted Pencil," *Proceedings of the London Mathematical Society* (1) 4, 1871-1873, pp. 337-341 (= *Scientific Papers*, vol. 2, pp. 332-337) and "On Hamilton's Characteristic Function for a Narrow Beam of Light," *Ibid.* (1) 6, 1874-1875, pp. 182-190 (= *Scientific Papers* 2, pp. 381-390); Joseph Larmor, "The Characteristics of an Asymmetric Optical Combination," *Ibid.* 20, 1888-1889, pp. 181-194 (= *Mathematical and Physical Papers*, vol. 1, pp. 181-193) and "The Simplest Specification of a given Optical Path, and the Observations required to determine it," *Ibid.* 23, 1891-1892, pp. 165-173 (= *Mathematical and Physical Papers*, vol. 1, pp. 264-270).

optics could only be profitable to the field. That this was the case is borne out by the fact that the Malusian approach received a lot of attention in many of the standard text books on optics of the time.

The best known example of the tendency towards a more abstract and theoretical formulation of the sciences in general is probably the Göttingen tradition incorperated by Felix Klein and, especially, David Hilbert. Broadly speaking, however, this tendency was visible throughout Europe.[64] Actually, this development had its roots in the French scientific scene of the early nineteenth century, precisely the scene in which Malus had unsuccessfully put forward his theoretical approach to geometrical optics. As explained at the beginning of this paper, at the time Malus' work did not have any impact. A century later, it finally would have. Whereas at the beginning of the century geometrical optics was not considered worthy of sustained attention in the French scientific community, at the end of the century it was to provide one of the most conspicious examples of the power of the abstract, mathematical formulation of physics. After more than a century, the abstract approach advocated by Malus had finally become commonplace.

From the nineteenth into the twentieth century

Before the 1870s, there seem to have been few connections between the various traditions in geometrical optics. The circles in which the technical opticians moved were simply too different from those in which the physiologists moved. In addition, the problems they were concerned with hardly overlapped. In the case of Malusian optics, the situation was similar. With the exception of Maxwell, who had a long standing interest in geometrical optics in all its aspects,[65] hardly any author on Malusian optics seems to have had any relations with either technical or physiological optics. Apparently, those who worked on Malusian optics were not even interested in phrasing their results in such a way that they might be useful to the construction of optical instruments or the study of the eye. Conversely, there was little interest in Malusian optics among the proponents of either technical optics or physiological optics. In general, they

[64] Cf. Christa Jungnickel and Russell McCormmach, *Intellectual Mastery of Nature. Theoretical Physics from Ohm to Einstein*, 2 vols. (Chicago, 1986). On the Göttingen tradition, see Mehrtens (n. 6), *Moderne-Sprache-Mathematik*.

[65] In fact, some of his very first publications were on geometrical optics. Cf. James Clerk Maxwell, "Solution of a Problem," *Cambridge and Dublin Mathematical Journal* 8, 1852, pp. 188-193 (= *Scientific Papers*, vol. 1, pp. 76-79) and "On the General Laws of Optical Instruments," *Quarterly Journal of Pure and Applied Mathematics* 2, 1858, pp. 233-246 (= *Scientific Papers*, vol. 2, pp. 271-285).

apparently were of the opinion that this kind of geometrical optics was unsuitable for the practical applications they had in mind.
As in the case of the development of geometrical optics at the beginning of the century, the factor of nationality has to be taken into account to explain this lack of contacts between the three traditions in geometrical optics. During the second half of the nineteenth century, for instance, the optical industry was concentrated in (Southern-)Germany and technical optics was very much a German affair. In France, the optical industry was barely developed and technical optics hardly studied. On the other hand, there was a more pronounced interest in Malusian optics than in Germany. Similarly, the mathematical study of the eye was an almost exclusively Scandinavian and Northern-German interest. In other words, different kinds of optics were concentrated in different countries or regions. This in itself would be sufficient to explain the lack of communication between the various parts of geometrical optics.

Later on in the nineteenth century, the nationality barriers had become considerably lower. With the advent of the second industrial revolution and the internationalisation of economy and industry, science also became more international. Although the extreme patriotism that was characteristic of the late nineteenth century could be found in science too, this should not obscure the fact that the exchange of scientific knowledge between the various European nations had only increased since the beginning of the century. In the development of geometrical optics as well, nationality became less and less important were the exchange of scientific knowledge was concerned. Accordingly, by the early 1890s, there was also an increasingly lively interaction between the three traditions in geometrical optics. Indeed, albeit for a short time, the different strands of geometrical optics even seemed to converge to one theory of geometrical optics. In 1891, Max Thiesen (1849-1936), a collaborator of the then recently founded *Physikalische Reichsanstalt*, published a paper in which he discussed the theory of optical aberrations with the help of mathematical techniques that are very similar to those developed by Hamilton in his optical work. A few years later, in 1894, the Leipzig astronomer Heinrich Bruns (1848-1919) formulated a more manageable version of Hamilton's optical theory and immediately applied this theory to technical optics.[66] In the same period, the physiologist and optician Gullstrand, whom I mentioned in the previous section, also developed a strong interest in Malusian optics. Central to their investigations was the study of systems of lines after they have been

[66] Max Thiesen, "Beiträge zur Dioptrik," *Sitzungsberichte der Königlichen Preussischen Akademie der Wissenschaften zu Berlin* 1890, pp. 799-813. Interestingly, the paper was presented by Helmholtz. On Bruns, see Johan Bauschinger, "Necrolog Heinrich Bruns," *Vierteljahresschrift der Astronomischen Gesellschaft* 56, 1921, pp. 159-169.

refracted a great many times or even an infinite number of times (as in the case of an optically inhomogeneous medium). For Thiesen, this question was of relevance because he wanted to consider optical systems that consisted of a great number of lenses. Probably, Bruns' work was related to a long-time interest he had in atmospheric refraction. A similar interest led Gullstrand to Malusian optics. In fact, Gullstrand wanted to study the optical properties of the crystalline lens of the eye. Basically, Thiesen, Bruns and Gullstrand resorted to the same kind of mathematics to solve their problem. Whereas Malus could make use of the newly developed Mongian geometry, all three of them utilised a variant of the theory of contact transformations that had been formulated by Sophus Lie in the 1870s.

During the early 1900s, similar investigations were pursued by many others, not only in Germany, but also in England and France.[67] In the course of these investigations, it soon became clear that one could deal with the transition of light rays through various homogeneous media in the same way as one could deal with the optics of inhomogeneous media. The key instrument in this case turned out to be the principle of least path, which we mentioned earlier when discussing Hamilton's work. As we have seen, there was a strong reticence to make use of this principle earlier in the century. Upon the admission of the principle of least path, however, Malus' optics of optically homogeneous media could in a very short time be transformed to the optics of discontinuous, inhomogeneous media. In fact, even media that were anisotropic, i.e., media in which the speed of light also depends on the direction of a ray were taken into account. By taking this very general approach, the early twentieth-century proponents of geometrical optics had considerably stretched Malus' definition of the field. Occasionally, the phenomena of light in inhomogeneous media had been considered before, notably in relation to the study of atmospheric refraction and that of the eye. With the exception of Hamilton, no one had ever studied the optics of these media in its full generality or on a systematic basis.

Probably, this extension of geometrical optics also has to be considered in the light of a growing interest in the relation between physical and geometrical optics. The exact expression of this relation which resulted from this interest made it clear how the extension of geometrical optics achieved at the end of the

[67] Cf. Lord Rayleigh (John Strutt), "Hamilton's Principle and the Five Aberrations of von Seidel," *London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science* (6) 15, 1908, pp. 677-687 (= *Scientific Papers*, vol. 5, pp. 456-464); Heinrich Weber, "Ueber den Satz von Malus für krummlinige Lichtstrahlen," *Rendiconti del circolo matematico di Palermo* 29, 1910, pp. 396-406; Théophile de Donder, "Sur les invariants intégraux de l'optique," *Bulletin de la Société mathématique de France* 42, 1914, pp. 91-95; René Dontot, "Sur les invariants intégraux et quelques points d'optique géométrique," *Ibid.*, pp. 53-91; Ernest Vessiot, "Sur les invariants intégraux de la propagation par ondes," *Ibid.*, pp. 142-167.

nineteenth century was the most complete that could be formulated. Any further extension would entail an appropriation of parts of physical optics.
A first sign of the renewed interest in the relation between geometrical and physical optics was the reconsideration of the role of the principle of least path and that of minimal principles in general in geometrical optics which occurred in the course of the second half of the nineteenth century. In the first decades of the nineteenth century, Malus had excluded such principles from geometrical optics. In the course of the last few decades, this exclusion began to be felt more and more as a limitation. Besides, minimal principles increasingly began to be considered as being of a mathematical nature. In the 1850s, the French mathematician Joseph Liouville (1809-1882) had prepared the way for the idea that from a purely mathematical point of view, the use of whatever minimal principle amounts to the determination of geodesics on higher-dimensional manifolds. In the following twenty years, this idea was developed to the full.[68] Therefore, minimal principles could be viewed as mathematical principles and there was no reason for excluding them from geometrical optics. Already in the early 1870s, in their studies on the infinitely thin pencil, Maxwell and Carl Neumann had strongly advocated the admission of minimal principles in geometrical optics, even though the actual use they made of these principles was of little effect. By propagating the use of such principles, however, they prepared the ground for their more conclusive application in the 1890s and 1900s.[69]

Another instance of the rise of a new view on the relation between physical and geometrical optics was provided by Ernst Abbe. In connection with his work for the optical firm Carl Zeiss at Jena, he expressed strong doubts about the adequacy of geometrical optics as a background to the theory of instruments. In particular, investigating the somewhat disappointing performance of a telescope he had developed for Zeiss, he had been forced to conclude that apparently the classical explanation for the functioning of a microscope was not correct. In fact, he had discovered that a microscope does not function in the same way as a telescope. Instead, he had found that its functioning was based on diffraction, i.e., on a phenomenon studied in physical optics.[70] This discovery had made it clear that the exact range of validity of geometrical optics still had to be

[68] See Jesper Lützen, *Joseph Liouville (1809-1889): Master of Pure and Applied Mathematics* (Springer, 1990), pp. 680-686.

[69] Cf. Maxwell (n. 65); Carl Neumann, "Ueber die Brechung eines unendlich dünnen regulären Strahlenbündels," *Berichte der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig. Mathematisch-physische Classe* 32, 1880, pp. 42-64.

[70] See Wittig (n. 44), *Ernst Abbe*, pp. 63-69, and the literature mentioned at the end of this book. For an account of the philosophical difficulties surrounding the discovery of the role of diffraction in the functioning of the microscope, see Ian Hacking, *Representing and Intervening. Introductory Topics in the Philosophy of Science* (Cambridge, 1983), pp. 186-209.

investigated. During the last few decades of the nineteenth century, Gustav Kirchhoff (1824-1887) and others tried to define this range of validity by viewing the foundations of geometrical optics as a limiting case of those of physical optics. In 1911, Arnold Sommerfeld (1861-1951) managed to give a mathematically exact expression of this idea. Essentially, what Sommerfeld showed was that by letting the wavelength converge to zero in the formulae given by Maxwell to describe physical optics, one obtains the formulae that Bruns used to describe geometrical optics.[71] This result obtained by Sommerfeld also suggested that with the extension of geometrical optics which had been formulated by Bruns, Gullstrand and others, the domain of this field had reached its 'natural' limits. Any further extension of the field would imply the incorporation of purely physical principles.

The First World War marked the culmination of the strong interest in the general geometrical optics of arbitrary media. The surge of investigations this interest generated led to a complete reformulation of the field. In the wake of this reformulation, technical and physiological optics would be subjected to a thorough transformation as well. In this connection, the influence of the war should be taken into account. In England and France, for instance, the study of modern geometrical optics was strongly promoted by the army.[72] It is probably no coincidence that the full institutionalisation of technical optics in these countries and the change in the approach to geometrical optics as a whole that went with it took place during the last years of the First World war. For this reason, the end of the Great War also marked the final acceptance of the abstract approach to geometrical optics as advocated by Malus. By 1918, the transition from the classical eighteenth-century mathematical tradition in optics to a more abstract and theoretically coherent geometrical optics as first proposed by Malus had finally taken place.

Whereas at the beginning of the nineteenth century, the study of the construction of optical instruments formed an impediment to the acceptance of

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[71] Arnold Sommerfeld and Iris Runge, "Anwendung der Vektorrechnung auf die Grundlagen der geometrischen Optik," *Annalen der Physik und Chemie* (4) 35, 1911, pp. 277-298, on pp. 289-293. On Sommerfeld's precursors, see Max von Laue, "Wellenoptik," in *Encyklopädie der mathematischen Wissenschaften*, Bd. 5.3, pp. 359-487, on pp. 437-439.

[72] For the situation in France, see for instance Emile Turrière, *Optique Industrielle* (Paris, 1920); this book reports on the results of two studies that were performed for the French army during the war. In England, Edmund Whittaker's *Theory of Optical Instruments* (Cambridge, 1908) was in such a demand by the army that it was reprinted in 1915 as vol. 7 of the relatively cheap *Cambridge Tracts in Mathematics and Mathematical Physics*; see H.A. Brück, *The Dunsink Observatory, 1785-1985* (Dublin, 1987), on pp. 178-179. Like Larmor and Rayleigh, Whittaker was a mathematical physicist; his main interest was rational mechanics. Turrière was a pupil of the Montpellier professor of mathematics Henri Bouasse.

the more abstract approach to geometrical optics as advocated by Malus, the development of technical and physiological optics over the nineteenth century formed an incentive to the acceptance of Bruns', Gullstrand's and Schwarzschild's equally abstract approach.

Concluding remarks

In the above I have shown how, at the beginning of the nineteenth century, Malus proposed a redefinition of the mathematical tradition in optics as the study of those phenomena that depend on geometry. Basically, this redefinition entailed the transformation of eighteenth-century dioptrics and catoptrics into the study of systems of straight lines and their reflection and refraction at single surfaces. Malus' approach met with little success. In fact, it was only at the beginning of the twentieth century, that geometrical optics was redefined in a more abstract form. This redefinition by Bruns and others a century after Malus was achieved by extending the field to the study of light rays in arbitrary media and consistently developing geometrical optics as a limiting case of physical optics.

In this paper, I have sketched how geometrical optics developed between Malus and Bruns. In order to facilitate the understanding of this development, I have included a diagram of the process (see fig. 1). In the course of the nineteenth century, geometrical optics diverged into three sub-fields: technical optics, physiological optics and what I called Malusian optics. Of these three, only Malusian optics was directly inspired by Malus' work and could be considered as an approach to geometrical optics in the spirit of the "Traité". The other two subfields not only principally drew their inspiration from pre-Malusian optics, but above all were also firmly embedded in other fields of study. In the case of technical optics, its problematic was as least as much determined by the practical requirements of the construction of optical instruments as by purely geometrical optical questions. Similarly, the development of physiological optics was intimately tied up with that of the physiology of the eye in general. Only by the end of the nineteenth century did these three subfields of geometrical optics converge to such an extent that one could talk about one field of geometrical optics again. This kind of geometrical optics could be considered as an example of a more abstract approach to the study of light rays as advocated by Malus. In its actual content, early twentieth-century geometrical optics had little to do with the kind of optics proposed by Malus. The scientific context in which geometrical optics had to operate was a completely different one from that of a century earlier and the topics to be studied had changed accordingly. In connection with the previous, the mathematics that was used in geometrical optics had changed in nature as well.

As I have shown above, Malus viewed geometrical optics as the study of two-dimensional systems of straight lines in space. For him, the eighteenth-century mathematical tradition in optics with its emphasis on the formation of images did not suffice any longer. Besides, his very geometrical approach to optics neatly tied in with Monge's attempts at a full geometrisation of nature. For the period after Malus, most of nineteenth-century geometrical optics in the Malusian tradition can be characterised by its marked insistence on the importance of systems of rays to the study of the phenomena of optics. Of course, even during the nineteenth century, it was realised that the study of two-dimensional systems alone might not suffice for all problems in geometrical optics. In the late 1830s, for instance, after almost fifteen years of work on geometrical optics, Hamilton had already come to the conclusion that the limitation to two-dimensional systems put a straitjacket on the development of geometrical optics as well. On the whole, however, within the Malusian tradition, the basic attitude towards geometrical optics as the study of systems of (rectilinear) rays rather than the study of the formation of images remained unchanged.

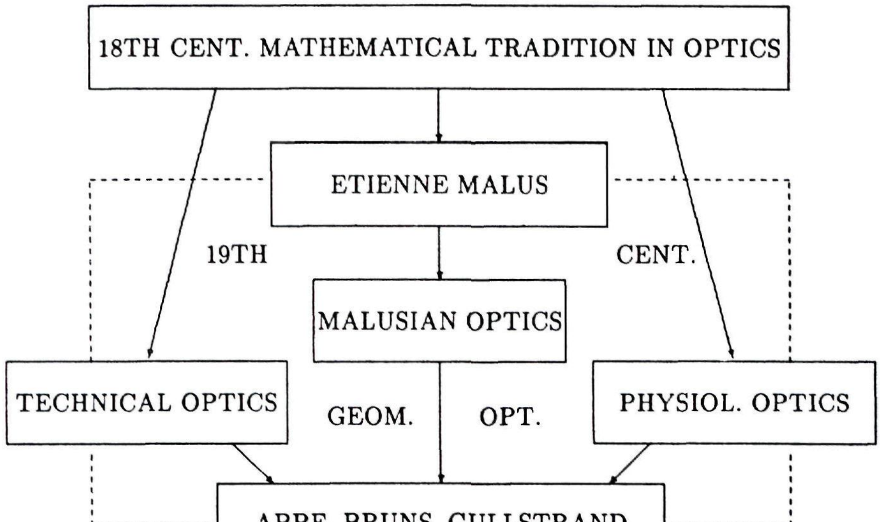

Figure 1 – Illustrating the development of nineteenth-century geometrical optics

Within the other two fields in geometrical optics, the importance of the study of systems of lines was less self-evident. In technical optics, it were not so much systems of rays as a whole that were important, but rather the individual rays. Accordingly, attention was focussed on methods to facilitate the determination of the paths of rays. The kind of mathematics that was used for these methods was analytical rather than geometrical in nature and mainly involved manipulation with Taylor expansions. Similarly, as in classical dioptrics and catoptrics, the determination of images remained an important topic in physiological optics. In line with the techniques utilised by Möbius, Gauss and Bessel, the mathematics that was used to deal with this topic involved the use of continued fractions and of what is now called matrix algebra.

By the end of the nineteenth century, the study of systems of lines gained new importance within the whole of geometrical optics. If we take Abbe, Bruns and Gullstrand as representative proponents of the three main approaches in geometrical optics, however, it is clear that these systems no longer were the two-dimensional systems of straight lines that had been studied by Malus. Reuniting and extending the goals of the three subfields in geometrical optics, they considered it to be the principal goal of geometrical optics to provide techniques to determine all possible paths of light rays in arbitrary media and to explain the formation of images through such media. In order to deal with this problem, it was of paramount importance to know more about the system of possible paths in one particular medium and the relations between such systems that are induced by the transition from one medium to another. The mathematics that was used in this case was basically Lie's theory of contact transformations. Expressed in modern terms, this theory can be characterised as a specific differential geometry of objects in six-dimensional space. The relation between two systems of possible paths in two different media induced by a transition from one medium into another then corresponds to a specific transformation of six-dimensional space that is now known as a canonical transformation. By specialisation, the theory of contact transformations not only provided a way to deal with the problems discussed by Malus, but could also be used to solve the problems central to technical optics as well as those important to physiological optics. Thus, the convergence of the three principal branches of nineteenth-century geometrical optics is also expressed on the purely mathematical level.

Summary

In the history of nineteenth-century optics a distinction has to be made between *physical optics* and *geometrical optics*. Whereas the first was concerned with the study of the nature of light, the second was about the study of the behaviour of light rays. In this paper, a sketch of the development of geometrical optics throughout the nineteenth century is given. It is argued that with Etienne Malus' "Traité d'Optique" of 1808 a new, more abstract, approach to classical dioptrics and catoptrics was proposed. Only with the work of Ernst Abbe, Heinrich Bruns and Allvar Gullstrand at the end of the century, however, did such a more abstract approach become commonplace and was the classical mathematical tradition in optics supplanted by another way of dealing with the phenomena of light. It is shown how this very gradual evolution of geometrical optics in the course of the nineteenth century was intimately tied up with the development of parts of the field to the full-fledged disciplines of *technical optics* and *physiological optics* and with the evolution of the relation between geometrical optics and physical optics.

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