STEVIN'S THEORY OF PERSPECTIVE:

THE ORIGIN OF A DUTCH ACADEMIC APPROACH TO PERSPECTIVE*

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Introduction

Since 1435, when Leon Battista Alberti presented his model for perspective drawing and the first description of a perspective construction, users of perspective have tried to find out why Alberti's and various others' perspective procedures led to a correct result. In the beginning the answers were based on mixed arguments, relying partly on intuition and empirical experience, partly on the optical theory of apparent sizes, and partly on geometry, especially the theory of similar triangles.

The first known example of an entirely geometrical deduction showing the correctness of a perspective construction occurs in Federico Commandino's comments on Ptolemy's *Planisphaerium* published in 1558. Three decades later, in 1585 to be precise, another example appeared in Giovanni Benedetti's *De rationibus operationum perspectivae* (Field 1985). Later, there was an essential change in the attempts to explain the perspective constructions and this gave rise to the creation of a mathematical theory of perspective. Where Commandino and Benedetti had concentrated on explaining some particular constructions, several of the mathematicians of the following generations approached the problem by searching for general laws for perspective projections.

This development was initiated at the very beginning of the seventeenth century in Italy and in the Netherlands by Guidobaldo del Monte and Simon Stevin respectively, who furthermore were both well experienced in both theoretical and applied mathematics. In 1600 Guidobaldo published *Perspectivae libri sex*; this is not an inviting work, for the proofs are long and the set-up is somewhat tedious. However, as the first book in the field of the mathematical theory of perspective it is admirable; in it Guidobaldo demonstrated that the mathematics of that time could solve all the problems of perspective to which a solution might be wanted

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– and many more. Moreover, it contains the seeds of many things which were further developed by later mathematicians. Stevin was the first to realize the possibilities of Guidobaldo's work, and only five years after its appearance published the elegant treatise *Van de verschaeuwing* whose content will be presented in sections two and three.

The deeper understanding of the theory of perspective also had consequences for the practice of perspective, because it implied that methods were found for determining perspective representations of lines of arbitrary directions. Before 1600 most of the artists' compositions were built up with four directions of lines, the verticals and three sets of horizontal lines defined in relation to the picture plane in the following way: the orthogonals, the lines that are parallel to it and those making an angle of 45° with it (Marcussen 1991). Other directions had been drawn, but not always correctly. Because the new theory of perspective treated all directions of lines it became possible for practitioners to represent these without mistakes.

Thus the new theory of perspective solved the old problem of why a construction is correct, and some of its results could be applied by practitioners. It was, however, in general impossible for practitioners to achieve mastery of the theory, despite the fact that many mathematicians – among them Stevin – expressly intended their books on the subject for them. The problem was that the mathematicians writing on perspective greatly overestimated the practitioners' capacity for following abstract mathematical deductions; this fact has made Laurence Wright characterize most of these writers as "dry-as-dust geometricians to whom drawing is an exact science" (Wright 1983, p. 158). No seventeenth-century book illustrates the problem of misjudging the practitioners' approach better than *Van de verschaeuwing*.

Seen from the point of view of transmitting knowledge to practitioners, Stevin's work was a failure, but with respect to creating a theory of perspective it was a great success – and considerably more successful than Guidobaldo's work. From the end of the eighteenth century and onwards Stevin's work on perspective has gained a fair amount of attention and praise in the literature (Poudra 1864, Wiener 1884, Papperitz 1910, Loria 1908, Struik in Stevin 1958, Sinisgalli 1978, Gericke 1990). Nevertheless it is possible to add to the understanding of Stevin's ideas by analysing his approach to perspective more closely, and by describing his solutions in terms of seventeenth-century concepts rather than by means of later concepts; the latter occurred especially in the early literature. Such an analysis is moreover a good starting point for a detailed investigation, not yet carried out, of Stevin's influence. Thus in this paper I shall first demonstrate how elegantly Stevin solved the fundamental problems of perspective; then I shall show that although *Van de verschaeuwing* did not become as well-known as it deserved to be, it influenced some Dutch mathematicians' to such an extent that it is appropriate to talk about a Stevin tradition in the Dutch academic approach to perspective; and finally I shall point to a minor influence of Stevin's ideas on some French textbooks.

The publication of Stevin's theory and its foundations

Before dealing with the content of *Van de verschaeuwing* I shall give an account of its printing history. It first appeared, as mentioned, in 1605; it was the first book of *Van de deursichtighe* (Illustration 1)[1] which again was the *derde stuck der wisconstighe ghedachtnissen* (third part of the mathematical thoughts). This work – consisting of five parts – was based on Stevin's lectures for Prince Maurice of Orange. Stevin used the word *deursichtighe* as a translation of perspective, and he applied it in the then traditional sense where it included some optical theories. He had planned that his work on the subject should consist of three books, one on *verschaeuwing*, one on catoptrics (*spieghelschaeuwen*), and one on refraction (*wanschaeuwing*). However, he never published the last book and made the second very short, so *Van de verschaeuwing* forms the main part of *Van de deursichtighe*.

Already in the year of its publication *Van de verschaeuwing* was translated into French and Latin by Jean Tuning and Willebrord Snellius respectively (Stevin 1605^2 and 1605^3) as part of their translations of Stevin's lectures for Prince Maurice. A couple of decades later Albert Girard prepared a more comprehensive French edition of Stevin's mathematical works, in which he included a revised version of Tuning's edition without making substantial changes in the treatise on perspective; the new edition was issued by Girard's widow in 1634. In this century Dirk Struik has reissued the major part of Stevin's mathematical works including *Van de verschaeuwing* together with an English translation, and Sinisgalli has republished the Latin edition of *Van de verschaeuwing* together with an Italian translation (Sinisgalli 1978).

Stevin was motivated to write *Van de verschaeuwing* because his student, Prince Maurice, after having been taught how to make perspective drawings by the "ablest masters of painting that could be obtained" asked him why perspective functions actually (Stevin 1958, p. 801). Concerning his reaction to the Prince's request, Stevin wrote:

> I perused and examined, more fully than before, several writers who deal with this subject and made a description thereof in my own words [in Dutch: *na mijn stijl*]. And after his *Princely Grace* had looked it through and helped to correct the imperfections that are commonly found in first attempts, had also fundamentally understood the common rule of finding the perspective of any given figure, and to his satisfaction practised it, I included this description among his *Mathematical Memoirs* ... [Stevin 1958, p. 801].

Unfortunately, Stevin did not reveal which authors he had studied. It would be particularly interesting to know whether he had read Guidobaldo del Monte's book before he published his own. On this question Struik wrote: "it is not unlikely that Stevin thoroughly enjoyed Del Monte's work. Despite this influence (which had to be inferred rather than proved by quotations) Stevin's work is an achievement of remarkable originality" (Stevin 1958, p. 790). I agree with Struik; nevertheless I will

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[1] For technical reasons most of the figures have been placed at the end of the paper; those occurring there are referred to as illustrations.

change Struik's "not unlikely" to 'more than likely', because there are so many of Stevin's expressions, theorems, ideas in proofs and constructions occurring in Guidobaldo's book that this cannot be accidental (see also Sinisgalli 1978, pp. 125-134). Irrespective of how much inspiration Stevin found in Guidobaldo's work he did however, as stressed by Struik and by himself, develop his own style. A style which is distinguished by being very precise and concise.

In his set-up Stevin followed the tradition of classical Greek mathematics and began with definitions and postulates; from these he deduced six theorems which constitute the foundation for his solution of perspective problems. Contrary to most of his predecessors he did not include anything from the theory of vision; thus for him perspective had become a purely geometrical subject. He opened *Van de verschaeuwing* with 16 definitions describing geometrical terminology. Stevin is known for having enriched the Dutch scientific language with numerous words, among them the word for mathematics – *wiskunde* (literally science of certainty, Dijksterhuis 1970, p. 128). Stevin also added several new words to the vocabulary of perspective, a few of which will be mentioned later on. In mathematicizing the situation of an observer looking at an object that has to be depicted on some material, he introduced the eye as a point (definition 6) and the material as a – transparent – infinite plane which he called the glass (definition 9); I frequently also use this term for the picture plane.

In his presentation of the concept of a perspective image Stevin's mathematical approach is very evident. His predecessors had introduced this concept in connection with plane or solid figures, but for a true mathematician like Stevin it was enough to introduce the perspective image of a point. He did so in his first postulate which states that a point, its image in the glass, and the eye are collinear. In his opinion this postulate did not define the image of a point which already lies in the glass, so he added a second postulate claiming that objects situated in the glass serve as their own images. Later, when van Schooten looked at Stevin's two postulates, he considered the second to be a corollary of the first.

In modern terms, Stevin's two postulates say that the perspective image in the glass of a point in space is the image obtained from a central projection that has its centre in the eye. In a comment Stevin explained that the complicated function of the eye made it necessary to postulate this relationship between the eye, an object point, and the image of the latter.

Before turning to the content of Stevin's six theorems I shall introduce two of his terms and their translations. To specify an object that has to be thrown in perspective Stevin used the adjective *verschauelick*, and he called the perspective image the *schaue*. In Snellius' Latin translation these words became *adumbrandus* and *umbra*, and in French Tuning and Girard similarly used the expressions *ombrageable* and *l'ombre*. The characterization of perspective images as shadows is not a very happy one, because many tracts on perspective – though not *Van de verschaeuwing* – deal with the specific problem of finding the perspective images of shadows cast by objects (a perspective image is also a rather peculiar shadow, because it lies between the object and the point of projection). In his translation Struik omitted the word *verschauelick*, and he used 'image' to translate *schaue*. I shall in general follow Struik, but when there is a need to make a clear distinction between an object and its image I shall – inspired by Brook Taylor – denote the first as the original object.

Stevin's first theorem, transcribed into the terminology just presented, states that the line segment joining the images of two original points is the image of the line segment between the two original points. Readers looking at Stevin's proof of this theorem may wonder why I have praised his precise mathematical style, for this is not demonstrated in his very first proof. He took it for granted that a line segment is depicted in a line segment, and said that it was clear that any point on the line segment between the original points will be depicted in a point lying on the line segment between the images of the two points. Although Stevin's proof is not very satisfactory, it is remarkable that he saw the need for stating this theorem concerning preservation of incidence between points and lines under a perspective projection. The dual theorem, that the point of intersection of the images of two original lines is the image of the point of intersection of the two original lines, is a result Stevin found so obvious that he did not state it explicitly. In his theorems he proceeded to investigate the perspective projection of sets of parallel lines – or rather line segments, because that is in general what he meant by lines. Thus in the second theorem he proved that parallel line segments which are parallel to the glass have images that are parallel line segments.

The third theorem is particularly important; therefore – and to give an impression of Stevin's style – I quote it in full in Struik's translation (the Dutch text is reproduced in note 2). To understand the quotation it is necessary to know that a 'ray' – often called a visual ray – means a line segment that has the eye as its one end point, and that the 'floor'means a horizontal plane of reference, often called the ground plane.

> Theorem 3
>
> If parallel lines are viewed through a glass that is non-parallel to the parallel lines, and their images therein are produced, they meet in the same point of the ray that is parallel to the parallel lines, and if the said lines are also parallel to the floor, their meeting point comes as high above the floor as the eye [Stevin 1958, p. 825].[2]

In modern terms theorem 3 states that a bundle of parallel lines that are not parallel to the glass is mapped into a pencil of lines, and that the point of convergence of the lines of the pencil is the point of intersection of the glass with that line of the bundle which passes through the eye. The topic of theorem 3 was treated by Guidobaldo in the five theorems 28-32 in the first book of *Perspectivae* (Guidobaldo 1600, pp 35-44). He started with a set of parallels lying in the ground

[2] The Dutch text reads: Evewijdeghe verschaeulicke linien ghesien sijnde deur t'glas dat onevewijdich is mette evewijdeghe, en haer schaeuwen daer in voortghetrocken wesende; sy vergaren in een selve punt des straels, dat evewijdich is mette verschaeulicke evewijdeghe, en de selve verschaeulicke oock evewijdich wesende mette vloer; haer saempunt comt soo hooch boven de vloer als het oogh [Stevin 1605, p. 17].

plane, and then proceeded first to a three-dimensional set of horizontal lines and secondly to an arbitrary set of parallels. Furthermore, Guidobaldo observed – like Stevin did in the end of theorem 3 – that when the bundle of parallels consists of horizontal lines then the point of convergence of their images is at the same distance from the ground plane as the eye. Guidobaldo used the term *punctum concursus* for the point of convergence, and this probably inspired Stevin to call it *saempunt*; in English Brook Taylor later introduced the technical term vanishing point, and that is the one I use.

Guidobaldo was aware that his result concerning vanishing points was important; apparently this made him think that it should also be carefully established, because he gave three different proofs for the same result. The first two are rather cumbersome, involving many similar triangles, but they contain all the necessary arguments – and a few more. The last proof is based on reflections about intersections of planes and lines; its idea is sound enough, but is not presented clearly. Stevin's proof of theorem 3 greatly resembles Guidobaldo's last proof; however, it is worked out more thoroughly (Illustration 2).

The proof of theorem 3, or its result combined with the observation in Stevin's second postulate that a point in the glass is its own perspective image, shows that the image of any given half line, having its end point in the glass and not being parallel to it, is the half open line segment joining the end point and the vanishing point of the half line (open at the end of the vanishing point). This result is one of the most useful theorems in the entire theory of perspective and has played an essential role in the development of this theory (Andersen 1984); because of its importance I call it the main theorem of perspective. Stevin did not mention it explicitly, but he applied it, as we shall see. He emphasized another consequence of theorem 3 which is actually a direct corollary, but Stevin thought it so important that he let it appear as a separate theorem, the fourth. This theorem asserts that the vanishing points of all sets of parallel lines that are horizontal and not parallel to the glass, have the same distance as the eye from the ground plane. One could have expected that Stevin would also have mentioned the line, now called the horizon, on which these vanishing points lie, but he did not (it is mentioned in Guidobaldo 1600, p. 45).

In treating the fundamental result concerning vanishing points Stevin used Guidobaldo's achievements as a base from which he could rise to a higher level, and in his further treatment of the theory of perspective Stevin worked fairly independently of Guidobaldo. The latter limited his considerations to picture planes that are perpendicular to the ground plane, whereas Stevin, as the first, wanted a theory that covered all positions of the picture plane. He was thereby led to a very ingenious observation concerning the invariance of a perspective image.[3] To be more explicit (Figure 1), let BCRS be a vertical glass, BC the intersection of the

[3] It is possible that in this case as well Guidobaldo served as a source of inspiration, because the starting point of Stevin's procedure is – as also mentioned by Rocco Sinisgalli – in Guidobaldo's work (Guidobaldo 1600, p. 150 and Guidobaldo 1984, p. 152 note 75). However, if Stevin started there he expanded the idea so much that the result is completely his own.

glass and the ground plane – commonly known as the ground line – E the eye and D the orthogonal projection of E on the ground plane, A a point in this plane and A_i its image in the glass; Stevin then considered the situation where the glass and ED are rotated simultaneously around BC and a line through D parallel to BC respectively, so that ED stays parallel to the glass. He wanted to know what happens with A_i under the rotation, and showed in theorem 5 that it will remain the image in the rotated glass of the point A seen from the rotated eye. Stevin's proof is fairly straightforward, not involving more than arguments concerning similar triangles. The impressive thing is not that Stevin proved the result, but that he thought of looking for it – an achievement which made Gino Loria write that Stevin had discovered "the fundamental theorem of the method of central projection" (translation from Loria 1908, p. 587), and moreover to name a more general theorem after Stevin (Loria 1907, p. 131).

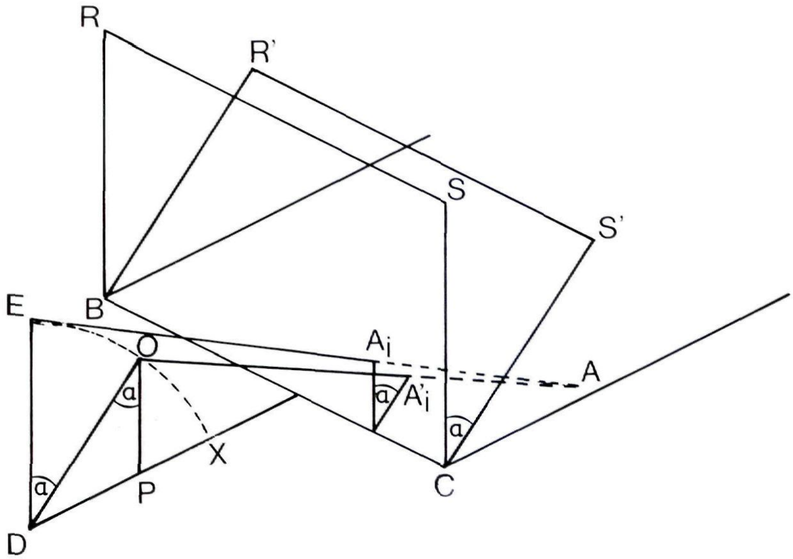

Figure 1 – A diagram illustrating Stevin's considerations concerning rotations.

Possessing the result concerning rotations, Stevin elegantly reduced the problem of constructing perspective images in oblique glasses to the one of constructing images in a vertical glass. His procedure was as follows. Let us assume that the glass BR'S'C makes the angle α with a vertical glass (Figure 1), and that a point in the ground plane seen from the point O has to be thrown in perspective in the oblique glass. All that is needed is to determine the point E which after a rotation of α degrees will fall in O. (The point E can be obtained in the following way: through P, which is the orthogonal projection of O into the ground plane, the line PX perpendicular to BC is drawn; on this line the point D is found so that the angle POD is equal to α, and finally DE is made equal to DO on the line through D parallel to PO.) Stevin also applied theorem 5 to derive a perspective construction in which the line XA – X being the point into which E falls when it has been rotated 90° – was used to find the image of the point A (Andersen 1989, p. 19).

The sixth and last theorem of *Van de verschaeuwing* deals with the same problem as theorem 5, but for a point above the ground plane; Stevin used this last theorem to construct images in oblique glasses of points lying above the ground plane.

In presenting Stevin's six fundamental theorems I have not quite followed the order of *Van de verschaeuwing*, for Stevin inserted some problems between the theorems. By collecting his problems I can better present the idea he had while selecting them, and I shall turn to this now.

The perspective problems in *Van de verschaeuwing*

The problems dealt with in *Van de verschaeuwing* fall into two categories. The one concerns proper perspective constructions, i.e., to find geometrical methods for determining the perspective image – in a given glass – of a given figure (whose position is also given) seen from a given point. The other category consists of a sort of inverse problem, namely that of determining, from a given perspective figure, the position of the eye when some information about the original of the perspective figure is given.

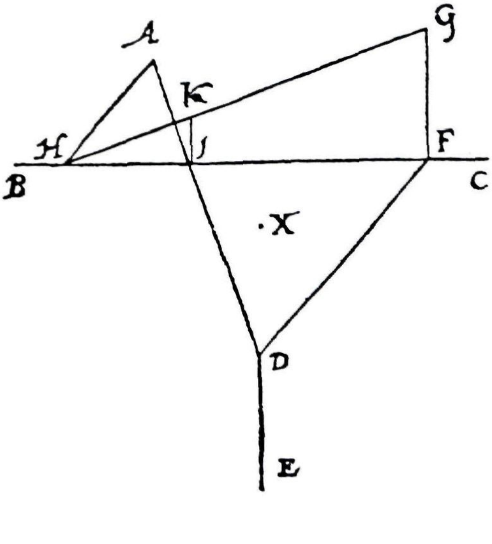

Figure 2 – Stevin's construction of the perspective image of a point in the ground plane. A missing C has been added to his drawing; Stevin 1605, p. 22.

Stevin decomposed the problems in the first group into the following basic ones: to find the image of a given point (lying either in the ground plane or above it) in a given glass (which can be either vertical, oblique or horizontal) seen from a given point. To illustrate his method I shall present his solution of the simplest case and then outline the way in which he derived the solutions for the other cases from this.

Let us then deal with the problem of finding the image of a point that lies in the ground plane when the glass is vertical (Figure 2). The line BC is the ground line, D the orthogonal projection of the eye on the ground plane, the height of the eye above this plan is given by the line segment DE, and finally A is the point in the ground plane whose image is requested. To follow Stevin's ideas it may be helpful to reproduce the three-dimensional configuration by rotating ED to the vertical position and by introducing a vertical glass (Figure 3); in Stevin's diagram the glass is turned into the ground plane so that it lies above the line BC (Figure 2). Stevin prescribed the following "operation" for finding the image of A in the turned down glass:

i Draw an arbitrary line DF that does not contain A, and determine the point F where it meets BC.
ii On the normal to BC at F make FG equal to DE.
iii Through A draw a line parallel to DF and find its intersection H with BC.
iv Draw the line GH.
v Draw AD and find the point I where it meets BC.
vi Draw the normal to BC at I and find its intersection K with GH.

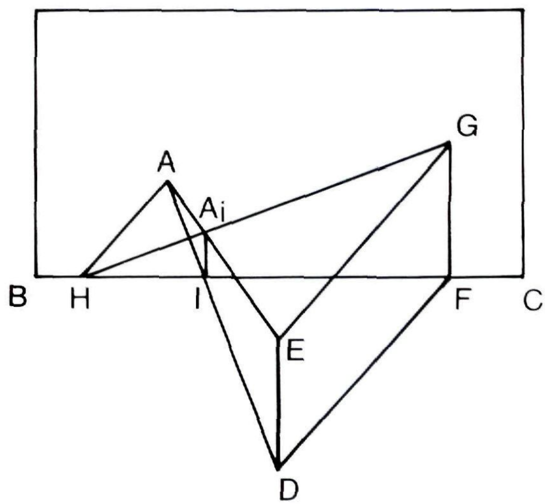

Figure 3 – The three-dimensional configuration corresponding to Stevin's drawing reproduced in figure 2; this figure is copied from Gericke 1990, p. 177.

Stevin then claimed that K is the required image, and proved it neatly. In paraphrasing his line of thoughts I introduce – to make things a shade clearer – A_i as the image of A (Figure 3) and then show that when A_i is rotated into the ground plane it coincides with K. From the fact that AH and DF are parallel and from the construction of G, follows that G is the vanishing point of the line AH; therefore A_i lies on GH (here Stevin applied the main theorem). Moreover since A_i is the point where the visual ray EA intersects the glass, and since DA is the orthogonal projection of this line into the ground plane, A_i lies on the normal to BC at the point, I, where DA meets it. Having found two lines in the glass – the normal and GH – containing the point A_i we can determine the latter as their point of intersection, and that is precisely how K was constructed.

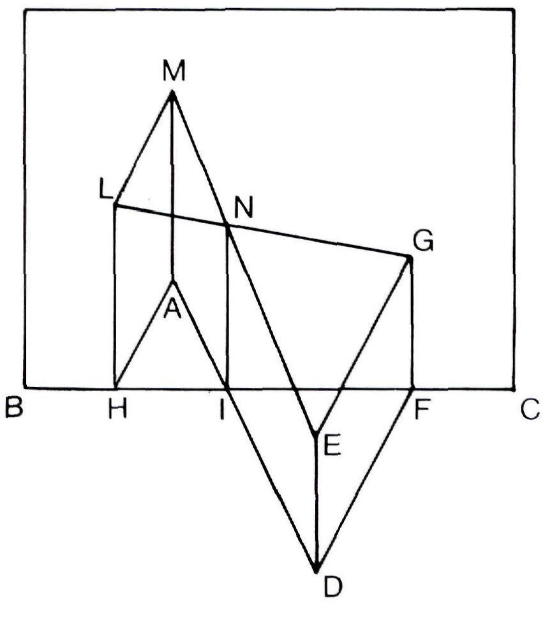

Figure 4 – The situation where the original point lies above the ground plane.

In book two of his *Perspectivae* Guidobaldo presented not less than twenty-three different constructions of the image of figures and in particular points in the ground plane; the tenth of these is the one which Stevin chose as his basic construction, and which he expanded to the general problem of determining the image of a point that lies above the ground plane. In presenting his solution for the latter problem Stevin introduced entirely new letters and thereby obscured the connection to the previous solution. To avoid this I shall keep the letters from Figure 2; the considered problem then becomes (Figure 4) to find the image of the point M, when its projection, A, into the ground plane and its height AM above this are given. Let ML be the line through M parallel to DF meeting the glass in L; the latter point can be determined by making HL equal to AM on the normal to BC. Arguments similar to those used in the previous construction show that the image, N, of M, is the point where the line GL meets the normal to BC at I. Thus the only difference between the constructions of the images of A and M is that in the first case GH is drawn and in the second GL (Figure 5).

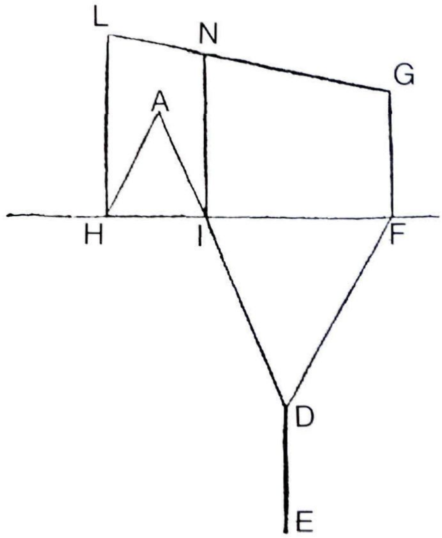

Figure 5 – Stevin's diagram, the letters are not original; Stevin 1605, p. 26.

Stevin's next step was to tip the glass; in the last section we have already seen how he reduced the problem of finding perspective images in an oblique glass viewed from a given point, to the one of determining an eye point for a vertical glass. Hence the solutions of the two first problems also provide the solutions for oblique glasses. To cover all cases Stevin also looked at the situation where the glass is parallel to the ground plane and the image of a point in the latter has to be determined. This problem he reduced to the first by introducing a new ground plane perpendicular to the glass and containing the given point. This solution is not so interesting, because the new ground plane can only be used for those points in the original ground plane that lie on the line of intersection of the two planes. Stevin knew that a plane figure situated parallel to the glass is depicted in a similar figure, but he did not mention this explicitly.

All Stevin's basic constructions were then reduced to the first (Figure 2) or its generalization (Figure 5). These constructions are remarkable, because they involve a rather unusual procedure. Like Stevin, many authors – before as well as after him – divided the drawing paper by a line corresponding to the ground line.

However, in general they placed the plans of the objects below the ground line and reserved the space above it for the perspective constructions. Geometrically speaking (Illustration 3i) this procedure corresponds – almost – to rotating the plan of the object into the picture plane. But not quite, because the described process produces the reverse side of the plan on the drawing paper (Illustration 3ii). Hence if the obverse side of the plan is used in the perspective construction the result will be a mirrored image (Illustration 3iii).

Some of the authors succeeding Stevin, for instance Charles Bourgoing, 's Gravesande and Brook Taylor, mentioned this problem, but most of them did not. Before Stevin nobody, as far as I am aware, touched upon the problem, which by the way was less relevant in the 15th and 16th centuries because the favourite objects in perspective compositions were symmetrical. It is difficult to say whether Stevin was worried about the mirror problem, and deliberately avoided it by choosing a solution which corresponds to turning the picture to that part of the ground plane which also contains the plan of the object (or to turning the plan upon the upper half of the picture plane Illustration 3iv), or whether he just took over this solution from Guidobaldo, who used it extensively. At all events, Stevin had obtained a solution which is mathematically satisfactory but rather inconvenient for practical use because the final perspective drawing is mixed up with the plan.

Having solved the problem of finding the image of a point regardless of how the glass is situated, Stevin had in principle solved all problems of perspective. He stressed this by demonstrating how his theory can be applied to constructing the perspective image of a rectilinear solid figure given by its plan and elevation – like the one reproduced in Illustration 4. Yet Stevin did not end his chapter on perspective constructions here; he knew that a point-wise construction is not the most refined solution for images of rectilinear figures. Hence he added a number of examples to illustrate how an application of his theory led to short cuts in the actual constructions. His examples mainly concern constructions of the perspective images of parallelograms and parallelepipeds lying in various positions in respect to the glass; an example is shown in Illustration 5.

In dealing with images of curvilinear figures a point-wise construction is the easiest solution, except in the cases where the image is a circle. Stevin revealed this insight by taking up the question "when will a circle be depicted in a circle?" This problem had earlier been treated by Commandino in his comments on Ptolemy's stereographic projections (Commandino 1558) and by Guidobaldo in theorem 28 of the fourth book of *Perspectivae*. The mathematics needed for answering this question occurs in Apollonius's first book on conics, more specifically in theorem 5 which states that if a section in a cone is a circle then the subcontrary section is also a circle (Illustration 6). Stevin showed how the ground plane and the glass can make subcontrary sections (Illustration 6); he dealt particularly with the situation in which the ground line is a tangent to the original circle (Illustration 7) and showed how the diameter of the image circle is found.

Stevin's further comments are another illustration of how a concrete problem stimulated him to search for more mathematical insights. First he suggested a means of checking the construction of a circle which is the perspective image of a circle; this is based on the tacit assumption that tangency is preserved under perspective projection and it consists of the following procedure (Illustration 7). Around the original circle a square is circumscribed; the perspective image of the latter is found; a necessary condition for the construction to be correct is that this quadrangular image is circumscribed about the image circle. Furthermore, the points of tangency in the perspective configurations have to be the images of the points of tangency in the original configuration. To this test Stevin added the remark that a circle inscribed in the image of the circumscribed square of an original circle must be the image of that circle, and that a circle inscribed in the image of a circumscribed rectangle of an ellipse must be the image of the ellipse. Although he did not pursue the matter further his considerations reflect ideas of looking projectively at conic sections.

The second part of *Van de verschaeuwing* dealing with inverse problems of perspective is, even more than the first part, a piece of theoretical mathematics. Stevin may have derived the idea of considering the subject from Guidobaldo, who treated three inverse problems of perspective, mainly because he needed their solutions for a particular perspective construction (Guidobaldo 1600, pp. 110-112). Yet although he created a very theoretical discipline, he could also have been inspired by a practical question, namely "from where shall one look at a perspective picture to perceive the scenery created by the artist?" Mathematically, this means reconstructing the eye point which the artist used for making his perspective drawing. Since the process of vision has an imaginary side it is often not so important to know the exact answer, but some perspective compositions give a stronger experience of a three-dimensional space when they are seen from the eye point, so a method for finding this is desirable. A universal method does not exist, because the general problem of finding the eye point for a perspective drawing has no unique solution. Such a solution can only be obtained when certain assumptions about the original objects are made. A typical example would be to assume that a tiled floor with horizontal rows of tiles is the image of a chess-board chequered floor.

Stevin was not particularly interested in solving the simple problems of inverse perspective, but was intrigued by the general problem. It is far from unlikely that he asked himself what information is needed to obtain a unique solution to an inverse problem. He did not find the answer to this question, which was actually not answered until the nineteenth century (Stevin 1958, p. 791), but he built up a system of more and more general problems. He began with finding the eye point when a quadrangle is given as the image in a vertical glass of a horizontal rectangle which has one side on the ground line, and when the ratio between the sides of the rectangle is given. Naturally enough Stevin solved this problem by inverting his method of perspective drawing (Illustration 8).

Proceeding systematically, Stevin reached his final two-dimensional problem, which treats a polygon with not less than four sides. The angle between the original polygon and the glass is given; furthermore, the original polygon is assumed to contain at least one pair of parallel lines, sides or diagonals, its shape (i.e., its angles and the ratios between its sides) is supposed to be known, and so is one of the angles which its sides form with the ground line. To come as far as solving this problem was quite an achievement; Stevin's technique will not be presented here, partly because an understanding of his solution requires familiarity with his previous problems and partly because the solution itself does not show any new sides of Stevin's theory of perspective. Stevin also touched upon an inverse problem concerning perspective images of three-dimensional figures, but he did not pursue the topic further than to treat one example. He completed the section on inverse problems by explaining why it is necessary to require that a polygon has at least four sides to obtain a unique solution.

The chapter on inverse problems of perspective is succeeded by a short section called *fautmercking* (detection of errors), which contains five rules of perspective useful either for avoiding or for detecting errors in perspective constructions. At the end of *Van de verschaeuwing* Stevin added an appendix in which he briefly discussed eight matters connected with perspective. I shall devote attention to the last three, the first of which is an interesting defense against the objection that some of the rules of perspective lead to unnatural results. In this defence Stevin once more demonstrated his superior understanding of perspective. The objection originates, Stevin showed, from a failure to distinguish between a reproduction of the visual impression of an object and a reproduction of the object that gives the eye the same visual experience as the object itself. He illustrated this by discussing an example which is very popular in the literature. Let us imagine (Figure 6) a person standing at C looking at a row of equidistant columns (placed in A, D etc.) and that he wants to depict them on a vertical plane parallel to the columns (indicated by the line HI). According to the rules of perspective the images of the columns will also be equidistant (the images being K, L etc.). This is by some considered unnatural, because the person at C will experience the distance FG as larger than the distance AD (since angle GCF > angle DCA); the idea is that the image of the columns ought to show the distances experienced. Stevin answered this argument by pointing out that in the perspective representation of the columns the visual angles are preserved and therefore also the relation between the apparent sizes. In

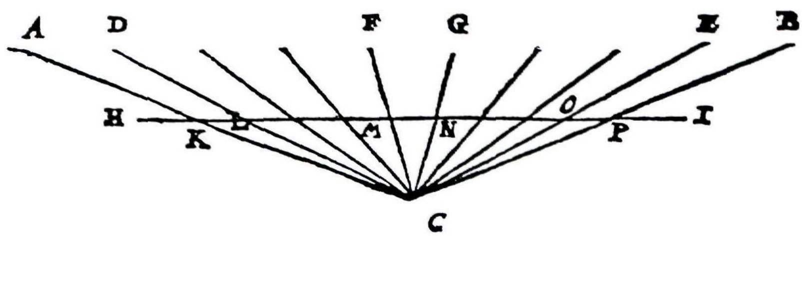

Figure 6 – A plan of a person looking at a row of equidistant columns; Stevin 1605, p. 83.

other words, the images of the columns will deceive the eye in the same way as the original columns do.

The next paragraph of Stevin's appendix contains the description of an instrument (Illustration 9) which he designed for Prince Maurice. Stevin said that he was inspired by a remark made, if he remembered rightly, by Albrecht Dürer. He might be referring to Dürer's comments in his *Underweysung der Messung* to the drawing reproduced in Illustration 10. The Prince had Stevin's instrument built and took great pleasure in it; among other things he used it to detect some mistakes in the manuscript to *Van de verschaeuwing.*

In the last paragraph Stevin calculated the sides and angles of a perspective square (Illustration 11). This is the only example where Stevin solved a perspective problem arithmetically. Thus it is difficult to decide whether it should be interpreted just as Stevin introduced it, namely as a response to the Prince's wish to make a calculation, or as a sign that Stevin had played with the idea of deriving formulas for perspective images. I do not consider the latter possibility to be unlikely, and if my hypothesis is correct, Stevin can be seen as a precursor of those mathematicians who later attacked perspective problems analytically. There was a particular interest in this approach in Germany in the second half of the eighteenth century, initiated by Abraham Gotthelf Kästner (Kästner 1752). However, it never became successful because geometrical constructions are much more convenient than calculations for solving perspective problems – at least before the age of computers.

The calculations end *Van de verschaeuwing.* It is really impressive how much material Stevin covered in its 92 pages (which in Girard's folio edition were reduced to 46). It is also impressive if, by reading Stevin's concise text, the Prince gained so much understanding of perspective as Stevin claimed (cf quote p. 27). In the next section I shall present others who were capable of understanding *Van de verschaeuwing.*

The influence of Stevin's work

Considering how elegantly Stevin had solved many of the fundamental problems of perspective it would be natural to expect that *Van de verschaeuwing* became a *chef d'oeuvre* in the theory of perspective. A work stimulating two developments, one that would result in a didactic explanation of the laws of perspective, and one that would add to Stevin's theoretical insights. However, things developed differently. Outside the Netherlands no significant reference was made to Stevin's work until 1837 when Michel Chasles wrote in his *Aperçue historique*:

> S'Gravezande et Taylor sont cités souvent, et à juste titre, comme ayant traité la perspective d'une manière neuve et savante: mais nous nous étonnons que l'on passe sous silence Stevin qui, un siècle auparavant, avait aussi innové dans cette matière qu'il avait traité en profond géomètre, et peut-être plus complètement qu'aucun autre, sous le rapport théorique [Chasles 1837, p. 347].

I share Chasles's surprise. The lack of recognition for Stevin's work cannot be explained by a language barrier, for besides being published in Dutch it was also available in French and Latin. Neither can it be explained by a lack of interest in the subject, because literature on perspective was flourishing after the appearance of *Van de verschaeuwing*. Stevin's style excluded most practitioners as readers, but should not have excluded the considerable number of seventeenth-century Belgian and French *savants*, who not only presented perspective constructions but also explained why they were correct.[4] The only – partial – explanation I can suggest is that the majority of those who wrote on perspective would search for books entirely devoted to the subject and would therefore not become aware of Stevin's work which was a part of his *Mémoires mathematiques*; the latter would more likely be studied by authors of general courses on mathematics. This explanation is in accordance with the circumstance that the only traces of Stevin's work I have found in the non-Dutch seventeenth-century literature do indeed occur in general works, and two of them in a *Cursus mathematicus*, one by Pierre Herigone from the 1630's and another by Claude François Milliet Dechales from 1674.

Herigone did not refer to Stevin's treatise, but there can be no doubt that he found some inspiration in it. Herigone used Stevin's idea of considering the picture plane as a glass and called it *le vitre*; he did not want to go into details with the theory, but presented the main results as axioms. Herigone's first axiom is similar to Stevin's first postulate (p. 28), and some of his later axioms contain material from Stevin's theorems. Moreover, Herigone's first construction of the perspective image of a point in the ground plane is the same as Stevin's (Herigone 1637, pp. 190-197). Contrary to Herigone, Dechales included the theory leading to the basic rules of perspective; his presentation of this theory is rather close to Stevin's, whereas he chose constructions different from Stevin's (Dechales 1674, vol. 2, pp. 465-532 or 1690, vol. 3, pp. 491-566). To the second edition of his *Cursus* Dechales added a section surveying the mathematical literature; in accordance with his approach to perspective he characterized *Van de verschaeuwing* here as a work that contains good demonstrations, but not a method adequate for the practice of perspective (Dechales 1690, vol. 1, p. 69). Between the appearances of Herigone's and Dechales's works the French scholar Mersenne had referred to *Van de verschaeuwing*; he used it, however, more as an excuse for not treating perspective in detail than as a source of inspiration. He thus copied a few of Stevin's results without repeating the proofs and without using drawings (Mersenne 1644, pp. 541-4).

The examples mentioned show that Stevin's work was not completely ignored in France; yet it had no real influence on developments there. In the Netherlands the situation was different; here *Van de verschaeuwing* was much better known, presumably because Stevin's mathematical works became part of the Dutch mathematical heritage. In fact I have come across only one Dutch pre-nineteenth-century mathematician who wrote on perspective and seemed to have been unfamiliar with *Van de verschauewing*; this is Stevin's contemporary and colleague

[4] Among these authors are François Aguilon, Jacques Aleaume, Charles Bourgoing, Etienne Migon, Jean François Niceron, Jacques Ozanam, Jacques Rohault, Andreas Tacquet and Jean Louis Vaulezard.

Samuel Marolois, whose book on perspective appeared nine years after Stevin's (Marolois 1614).

An illustration of the familiarity with *Van de verschaeuwing* is found in a letter which Stampion de Jonge wrote to his young pupil Christiaan Huygens in 1645. In this de Jonge listed various mathematical disciplines as study objects for Huygens, among them perspective which, he said, could be learnt by reading one of the three authors: Jan Vredeman de Vries, Marolois, or Stevin (Huygens *Oeuvres*, tome I, p. 6). A letter which Christiaan wrote to his brother Constantijn twenty-three years later reveals that he decided to read Stevin's work: "Pour la perspective je n'ay veu aucun autheur que Stevin, c'est pourquoy je ne puis vos dire qu'il est le meilleur" (Huygens *Oeuvres*, tome VI, p. 46). The letter also shows that Huygens did not acknowledge the need of text books on perspective: "Il y a si peu difficulté en cette science qui se peut comprendre dans une ou deux regles, que je ne doute pas que vous ne puisserez trouver tout par vous mesme. Dites moi ce que vous y trouvez de plus difficile et je vous l'expliqueray" (*idem*).

This sophisticated attitude had the surprising consequence that Huygens paid no attention to *Tractaet der perspective* (Illustration 12) which was written by his former mathematical mentor, Frans van Schooten, with whom he had had a close scientific contact. The fact that this treatise appeared in van Schooten's *Mathematische Oeffeningen* (1660) together with one of Huygens's own works – his famous one on calculating chances, *Van rekeningh in spelen van geluck* – makes Huygens's ignorance of van Schooten's work even more striking. A reference to this tract, published eight years before Huygens wrote to his brother, would have been a good answer to Constantijn's question, because it is a very commendable introduction to the theory of perspective, at which we shall now take a closer look.

Like Stevin, van Schooten did not refer to any other writers on perspective, but he was without doubt very much influenced by Stevin or, to put it more strongly, had Stevin's text before him when he wrote his own. He took over Stevin's set-up and most of his terminology, and he used his definitions, postulates, theorems and problems. The result was no mere plagiarism, but rather a second, and in general improved, version of the introductory part of *Van de verschaeuwing*. I find it interesting to see how van Schooten, who is known for his wish to transmit mathematical knowledge, first of all the richness in René Descartes's *La géométrie*, dealt with Stevin's insights, hence I shall describe the substance of the changes he made in *Van de verschaeuwing*.

Van Schooten did not completely agree with Stevin about what should be called definitions and what should be called postulates and made it, for instance, a postulate that the eye is a point. As mentioned earlier, he found Stevin's second postulate superfluous and made it a corollary to Stevin's first postulate which he took over (p. 28). In his revision of the theory van Schooten rearranged Stevin's first four theorems – making five out of them – so that they appeared in a more pedagogical way (Illustration 13), and he exchanged Stevin's proof of theorem 1 with a new and more satisfactory proof (p. 29). Moreover, he left out Stevin's two theorems concerning rotation of the glass and the eye (p. 31), but added a sixth theorem dealing with a plane figure parallel to the glass; a subject which Stevin had not treated.

In selecting problems van Schooten opened with the very important one of determining the images of a lines that are not parallel to the glass. His solution is equivalent to the main theorem (p. 30); this theorem does not occur explicitly in Stevin's work, but it is formulated as a problem in Guidobaldo's book (Guidobaldo 1600, p. 44). There are a few more examples where van Schooten included considerations which were published by Guidobaldo and not by Stevin; thus it seems that van Schooten used Guidobaldo as a supplementary source of inspiration. From the image of lines van Schooten proceeded to constructions of perspective images of points; he applied the same way of turning the glass into the ground plane as Guidobaldo and Stevin, but he made a small change in Stevin's first construction by building more on the main theorem. Thus, instead of using an arbitrary line through a given point and the projection of the visual ray (DA in Figure 3), van Schooten chose two arbitrary lines through the point (Illustration 14); this is a rather obvious application of the main theorem which was also made by Guidobaldo (Guidobaldo 1600, p. 72). In considering the positions of the original points van Schooten was, for once, more general than Stevin; thus he also looked at the situations where the original points lie between the eye and the picture plane and where they lie below the ground plane. Like Stevin, van Schooten wanted to treat problems concerning images of points in an oblique glass; he did, however, not want to take over Stevin's solution which involved rotations, and Guidobaldo offered no solution, so he created his own (van Schooten 1660, pp. 533-539).

Van Schooten's aim with *Tractaet der perspective* was to present the fundamental theory of perspective; thus he stopped at the constructions of images of points. But he did lecture on the practice of perspective; it would be interesting to know how he applied his theory. There might be a possibility of getting this information, because his brother Petrus van Schooten took notes at the lectures (van Schooten 1660, p. 523). Unfortunately I have not yet succeeded in tracing these notes.

While writing *Tractaet der perspective* van Schooten had practitioners in mind, because they had, he thought, a great need to learn their discipline from the correct foundations, and he wanted to give them a clear introduction to these (van Schooten 1660, p. 502). With respect to reaching the practitioners it is doubtful whether he succeeded, because it requires more familiarity with geometrical proofs to understand his treatise than the common users of perspective possessed – and possess. Nevertheless, by leaving out many of Stevin's mathematical speculations van Schooten had made it easier to realize what fundamentally was required to understand the principal rules of perspective. He did succeed in presenting the theory clearly; that he managed so well is undoubtedly due to the fact that he had such a good model. Considering how close he kept to this it is remarkable that he did not acknowledge his debt to Stevin. References in mathematical tracts on perspective are on the whole rare phenomena; a fact that complicates the art of determining historical development. Thus far I have proved that there is a link between Guidobaldo and Stevin, and that van Schooten built upon Stevin. In pursuing the further history of the Stevin tradition I have made linguistic comparisons and looked upon choice and proofs of theorems. I shall indicate some of my arguments, but to avoid becoming too repetitive I shall not go into details.

Van Schooten provides an example of what I looked for in the beginning of this section, an author who profited by the richness of Stevin's *Van de verschaeuwing*. He translated parts of his *Mathematische Oeffeningen* into Latin, but not his treatise on perspective, so this very clear introduction was reserved for readers who knew Dutch – and, as I have argued, mathematics. Thus the group of contemporary potential readers of *Tractaet der perspective* was rather small, leaving an exclusive group of actual readers. One of these was the Danish mathematician Georg Mohr, who spent several years in the Netherlands. Here Mohr worked out the idea of performing traditional geometrical constructions without the use of a ruler. He presented his result in the book *Euclides Danicus* which appeared in 1672 in two editions – a Dutch and a Danish; neither of these was noticed, so for a long time the Italian mathematician Lorenzo Mascheroni was credited for being the first to realize that Euclidean constructions only require the use of a compass (Hjelmslev 1931). Most of the construction problems occurring in *Euclides Danicus* are taken from Euclid's *Elements*; four of them, however, concern perspective. Mohr's solutions to these are based on the construction of a fourth proportional and do not resemble the constructions by van Schooten. However, Mohr took the problems and his vocabulary from van Schooten; an example is shown in Illustration 14. The texts reproduced there show why I mention van Schooten rather than Stevin as Mohr's source of inspiration. It can be noticed especially that Mohr took over van Schooten's expression *teyckentlick* to characterize an original object and not Stevin's term *verschaeulick*.

The next trace of the Stevin tradition I have found occurs in Abraham de Graaf's *De geheele mathesis* (the entire mathematics). This work was first published in 1676 and became so popular that it went through at least seven editions. Having to deal with all mathematical disciplines in one work, de Graaf had to be concise; in only ten pages he covered a presentation of the theory of perspective and some basic constructions – and he did it well, helped by not less than 117 figures (de Graaf 1694, pp. 213-222). The theoretical part consists of seven theorems which are van Schooten's six theorems rearranged. De Graaf added, however, a new observation to Stevin's theory, namely that a point on a line in an infinite distance will be depicted in the vanishing point (he did not use that term) of the line. It was only the introductory theorems of van Schooten's treatise of which de Graaf approved so much of that he used them; when it came to constructions he chose some different from van Schooten's, even some rather unusual ones.

The story about the influence of Stevin's theory continues with the book *Verhandelingen van de grontregelen der doorzigtkunde* (treatise on the fundamental rules of perspective) which was published in 1705. In this the theory of perspective and some perspective constructions are dealt with in such a way that there can be no doubt that de Graaf's book had inspired the author. The latter was Hendrik van Houten, about whom I have been unable to find any further information. Judging from the content of his book I suspect that van Houten was well acquainted with the perspective literature published by practitioners. This mainly consists of manuals containing some brief descriptions – without mathematical explanations –

of some rules of perspective and for the rest a wealth of examples. There were different traditions for selecting the examples, but in all cases the idea was to show more than just the way in which simple geometrical figures are thrown in perspective, for instance how a staircase, a complete room or an exterior look in perspective. In his choice of compositions van Houten seems to have been inspired by the Dutch tradition initiated by Vredeman de Vries as well as by an Italian tradition – and maybe also by Marolois. More important, however, than where van Houten found his inspiration is the fact that he managed to combine the theoretical and practical approaches to perspective, a rare occurrence in the history of perspective. The separation between the two approaches was often regretted by both mathematicians and practitioners, but very few were able to bridge the gap.

One of those who criticized the theoretical books on perspective for paying too little attention to practice, and vice versa, was the mathematician Willem Jacob 's Gravesande. In an attempt to improve the situation he wrote *Essai de perspective* which was published in 1711.[5] Although 's Gravesande showed great concern for finding applicable constructions, he kept so much to the style of his discipline that his book appeals more to mathematicians than to practitioners. From a theoretical point of view, however, his contributions are interesting and have been acknowledged as being important in the history of mathematics. The main part of 's Gravesande's achievements are not directly connected to Stevin's work so I do not present them here but concentrate on his introduction of the foundation of perspective. He himself stressed that:

> Ce qui a été découvert de plus utile sur cette matiére s'y trouve réduit à trois Théorémes généraux, sçavoir le premier, le second & le quatriéme; toute le reste s'en déduit par voye de Corollaire. A ces Théorémes déja connus ... ['s Gravesande 1711, Preface, pp. 8^v-9^r].

The theorems mentioned are indeed well-known, because they are another edition of Stevin's fundamental theorems, keeping the order Stevin employed and taking in van Schooten's revisions. 's Gravesande made the change to let the main theorem occur instead of theorem 3, from which Stevin and van Schooten had deduced it, the first implicitly and the second explicitly.

As far as I am aware *Essai de perspective* is the last Dutch book in the Stevin tradition written while the theory of perspective still had its own life; the latter came to an end in the nineteenth century when the subject first became part of descriptive geometry and later of projective geometry. The traces of the Stevin tradition do not however stop with 's Gravesande because he exported, so to speak, the insights of the Dutch mathematicians to the other side of the Channel. There the English mathematician Brook Taylor became aware of the fruitful Dutch approach to the theory of perspective. In 1715 Taylor published – besides his

[5] David Bierens de Haan wrote that in 1707 's Gravesande published a *Proeve over de doorzigtkunde* (Bierens de Haan 1883, p. 111). After much searching and after having consulted Jan van Maanen and Kees de Pater – to both of whom I am grateful for their kind help – I have come to the conclusion that such a book does not exist; no manuscript of it is known, either.

famous *Methodus incrementorum* which gave birth to the Taylor series – *Linear Perspective* and in 1719 *New Principles on linear perspective*. The introductory parts of these two books show clear signs of inspiration from 's Gravesande (Andersen 1989, pp. 20-23). The books are mathematically more advanced that Stevin's *Van de verschaeuwing*; nevertheless they inspired English practitioners to unify the theoretical and practical approaches to the discipline. Thus the practitioners included geometrical proofs of the rules of perspective, gave vanishing points an important role, and benefitted from the main theorem – for which they gave Taylor the credit. However, this is another story which, although it has a slight connection to Stevin's influence, will not be told here (Andersen 1989, pp. 52-59, 63-66).

Before summing up I shall briefly touch upon the question of whether there was a development in the Netherlands similar to the one in England, or in other words whether Stevin's theory influenced the Dutch practical literature on perspective. The answer is that there is no sign of a greater influence, but that van Houten's work provides at least one example of combining the practice of perspective with Stevin's theory (p. 44). The rest of the Dutch literature I know of, which treats the practice of perspective according to practitioners' tradition, does not contain any theory.[6] One minor exception is a manual written by Jacob de Vlaming (de Vlaming 1773); at the end of this Vlaming addressed himself to the readers who might be interested in knowing the theory behind the procedures presented by him, and he then listed four of van Houten's theorems – without reference or proofs.

Concluding remarks

My hope is that the present paper has made it evident that the story of Stevin's contribution to the theory of perspective provides an interesting and relevant chapter of the history of mathematics. The story shows, among other things, that the problem of understanding the rules of perspective, a troublesome one for almost two hundred years, found an elegant solution when a gifted mathematician took an interest in it. The story also illustrates how a mathematician taking his starting point in an explicit problem is seduced by the mathematics and turns to mathematically fascinating, but not immediately applicable generalisations.

The story of Stevin's influence is slightly surprising. One could have expected that his goal of having geometrical arguments introduced into practitioners' presentation of the basic rules of perspective had been reached gradually in the Netherlands. Furthermore, it could have been expected that mathematicians

[6] In the list of literature I have included those books known to me that are entirely devoted to perspective and treat the subject according to the practitioners' tradition. The authors of these books are Dirk Bosboom, Hendrik Hondius, Caspar Jacobszoon Philips, Jacob de Vlaming and Jan Vredeman de Vries. In a forthcoming paper I shall survey the Dutch pre-nineteenth-century literature on perspective.

succeeding him would have been inspired by his more theoretical investigations, for instance on inverse problems of perspective. What happened, however, was that in general his attempt to revise the approach of the practitioners failed, and that the mathematicians took up that part of his theory that was aimed at the practitioners and left the rest untouched. The latter may partly be explained by the fact that his successors who wrote longer mathematical treatises on perspective, van Schooten and 's Gravesande, addressed themselves to the practitioners.

In the actual development Stevin's achievement with respect to the foundation of the theory of perspective was an important one. It enabled the Dutch mathematicians in the seventeenth century and the beginning of the eighteenth to treat perspective clearly and elegantly; 's Gravesande especially profited from Stevin's work in creating his own methods of perspective constructions.

Stevin's work is, however, not only remarkable in connection with the Dutch development. The real progress in the entire history of the mathematical theory of perspective was made by a few mathematicians, among whom the most outstanding are Guidobaldo, Stevin, 's Gravesande, Taylor and Johann Heinrich Lambert.[7] One of my interests has been to find connections between these scholars, and I hope to have convinced the reader that Stevin played an important role in a continuous development from Guidobaldo to Taylor.[8]

Summary

In 1605 Simon Stevin published a remarkable treatise, *Van de verschauewing*, in which many basic mathematical problems concerning perspective representations are solved elegantly. The present paper surveys the content of the work and discusses its role in the development of the theory of perspective from Guidobaldo del Monte to Brook Taylor. It is shown that Stevin was inspired by Guidobaldo, but found his own style which became influential for the Dutch academic treatment of perspective. Among the followers of Stevin were Frans van Schooten and Willem 's Gravesande, in the work of the latter Taylor found some stimulation.

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[7] Some readers might have expected to find Girard Desargues's name in this list. However – I shall give more details in a forthcoming paper – Desargues's tract on perspective contains no theory, and his theory of projective geometry does not deal with perspective (cf. Field 1987).

[8] Whether there is a link between Taylor and Lambert cannot be definitely decided; I am, however, inclined to believe that Lambert's work is independent of Taylor's. (Andersen 1989, pp. 24-25).

DERDE STVCK DER WISCONSTIGHE GHEDACHTNISSEN VANDE DEVRSICHTIGHE.

De perspecti-ve.

Inhoudende t'ghene daer hem in gheoeffent heeft

DEN DOORLVCHTICHSTEN
Hoochgheboren Vorſt ende Heere MAVRITS Prince van
Oraengien, Grave van Naſſau, Catzenellenbogen, Vianden, Moers &c.
Marckgraef vander Vere, ende Vliſſinghen &c. Heere der Stadt Graye,
ende S'landts van Cuyc, S[t]. Vyt, Daeſburch &c. Gouverneur van
Gelderlant, Hollant, Zeelant, Weſtvrieſlant, Zutphen,
Vtrecht, Overyſſel &c. Opperſte Veltheer vande
vereenichde Nederlanden, Admirael
Generael vander Zee &c.

Beſchreven deur SIMON STEVIN *van Brugghe.*

TOT LEYDEN,
By Ian Bouwenſz. woonende op de hoogelantſche Kerckgraft.
Anno CIↃ IↃ CV.

Illustration 1 – Title page of the work containing *Van de verschaeuwing.*

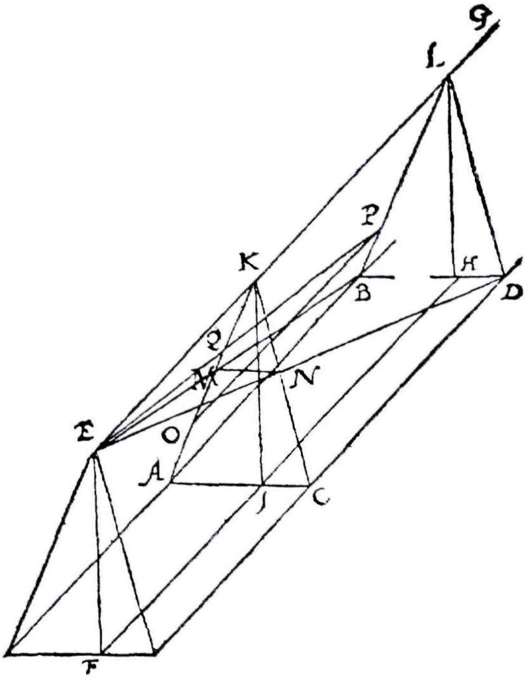


Illustration 2 – Stevin's diagram to his third theorem. To prove this Stevin considered a line segment AB which, extended, does not pass through the eye and which has an endpoint in the glass. He then showed that its image, AM, lies on the line of intersection of the glass ACK and the plane defined by the eye and the line segment AB. Furthermore, he showed that this line of intersection contains the point K, where the line EG parallel to AB meets the glass. Thus AM, extended, will pass through K; since K is only dependent of the direction of AB Stevin concluded that the image of any line segment parallel to AB will, extended, pass through K. Hence K is a point of convergence, or in modern terms the vanishing point of all lines with the same direction as AB. In his proof Stevin only considered line segments that have an end point in the glass, but he could easily have finished the proof of his general statement about any set of parallel line segments by referring to theorem 1; Stevin 1605, p. 18.

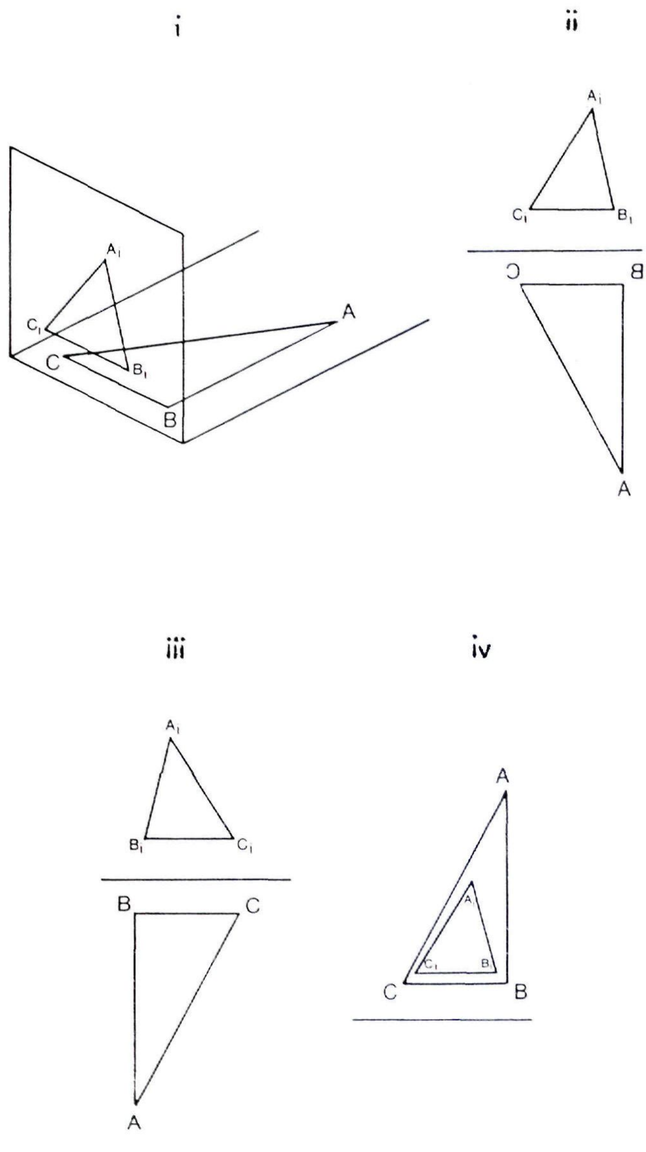


Illustration 3 – Diagrams concerning various ways of bringing the picture and the plan in the same plane.

i The three-dimensional situation.
ii The plan rotated into the picture plane.
iii The result of using the obverse side of the plan.
iv The picture plane rotated into the ground plane to the side of the plan.

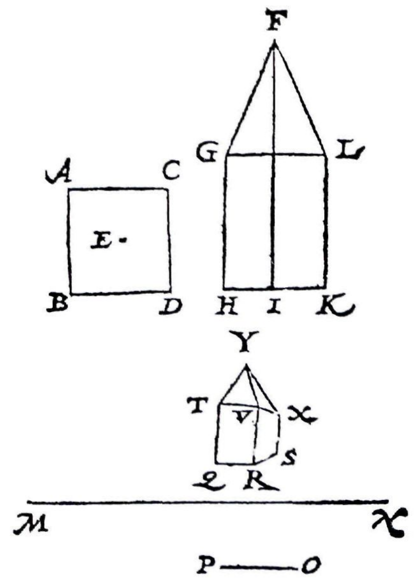


Illustration 4 – Stevin's construction of the perspective image QRSTVXY of a tower, whose plan is ABCD and whose elevation is FGHKL; P is the projection of the eye on the ground plane and PO marks the distance to the eye (Stevin 1605, p. 36). In this example the glass is supposed to be vertical; Stevin also showed how the images of the tower look when the glass is oblique, inclined either towards the ground plane or towards the eye, and when the glass is horizontal.

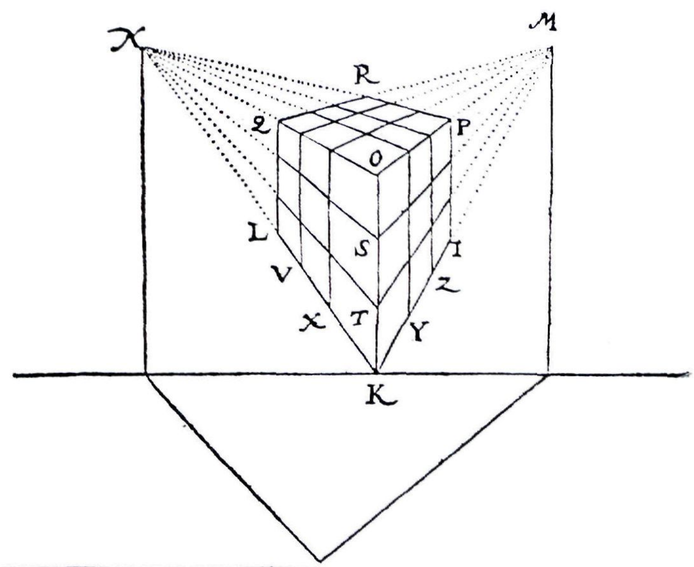


Illustration 5 – A cube thrown into perspective; Stevin 1605, p. 55.

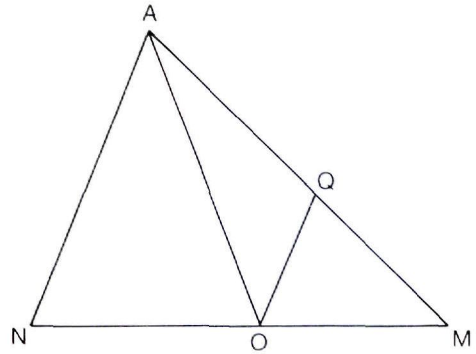


Illustration 6 – Let AOM and AOQ be axial triangles in a cone, and let OM and AQ determine two sections perpendicular to the triangles. If the triangles are similar, the sections are said to be *subcontrary*. If the section determined by OM lies in the ground plane and is a circle, and if the plane determined by OQ is conceived as the glass and A as the eye point of a perspective projection, then the circle with the diameter OM will be depicted in the glass as a circle with diameter OQ (illustration 7).

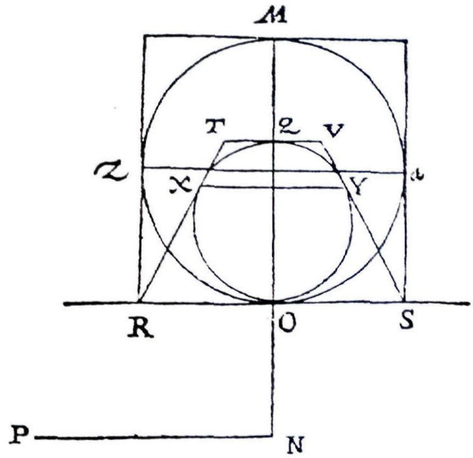


Illustration 7 – Stevin's diagram showing how an original circle with diameter OM is depicted into the circle with diameter OQ. The line RS is the ground line, N is the point where a line through the eye parallel to the glass (line AN in illustration 6) meets the ground plane, NP marks the distance to the eye, furthermore the slope of the glass eguals angle QOM in illustration 6; Stevin 1605, p. 59.

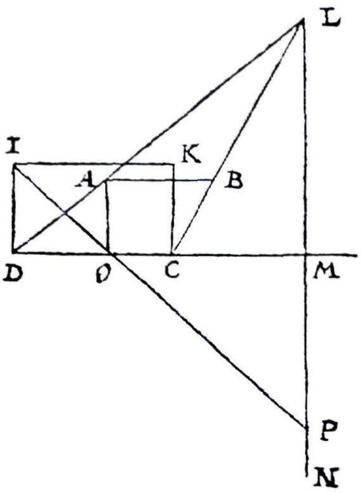


Illustration 8 – Determination of the eye for the perspective rectangle ABCD where CD lies on the ground line, the ratio between the originals of DC and AD being given. The point L where AD and BC meet is the vanishing point of the orthogonals. Hence L gives the sideward position of the eye and LM its height, M being the point of intersection of the ground line and the normal to it through L. On DC is constructed a rectangle DCKI congruent to the original, the point A is projected into the point O on DM, IO is drawn meeting LM in P, the latter point is the projection of the eye on the ground line (cf. the construction in figure 2); Stevin 1605, p. 61.

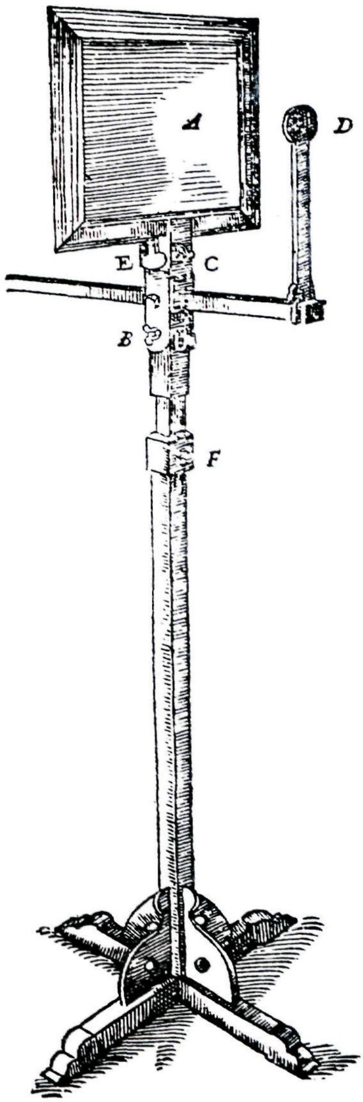


Illustration 9 – Stevin's instrument; Stevin 1605, p. 89.

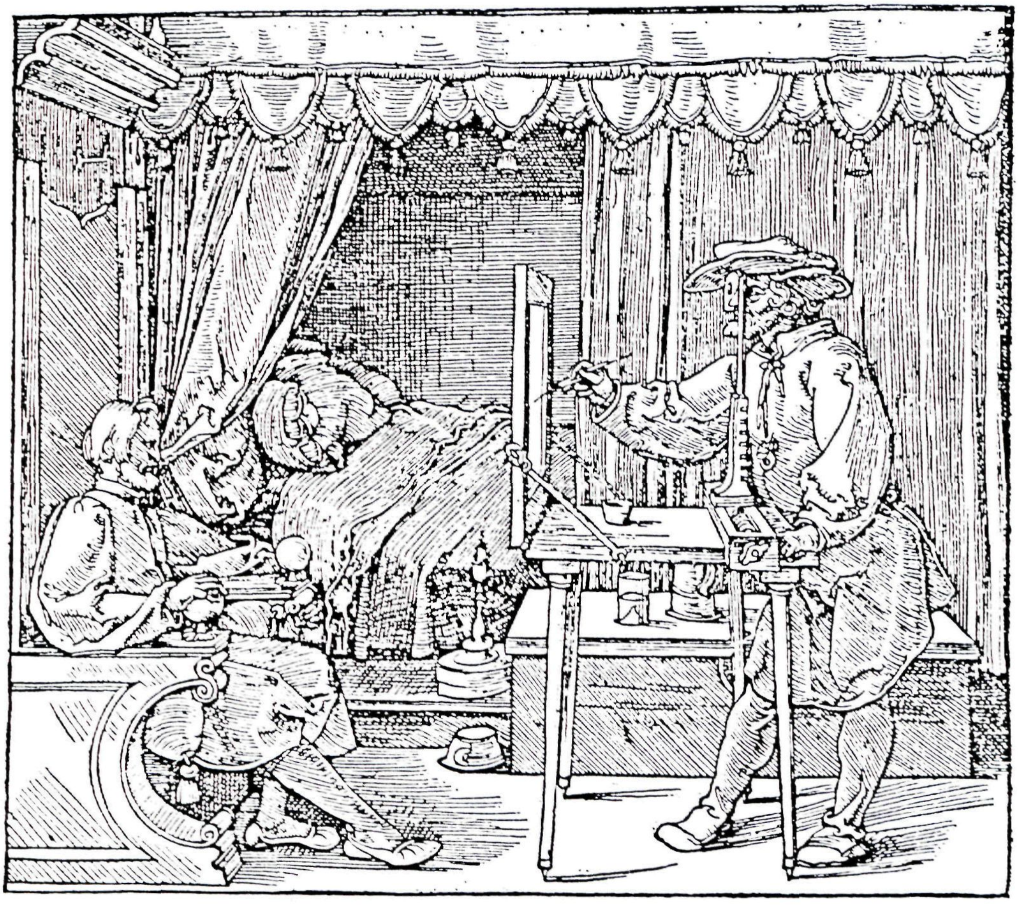


Illustration 10 – One of Dürer's instruments for making perspective drawings; Dürer 1525, p. Q ii^{v}.

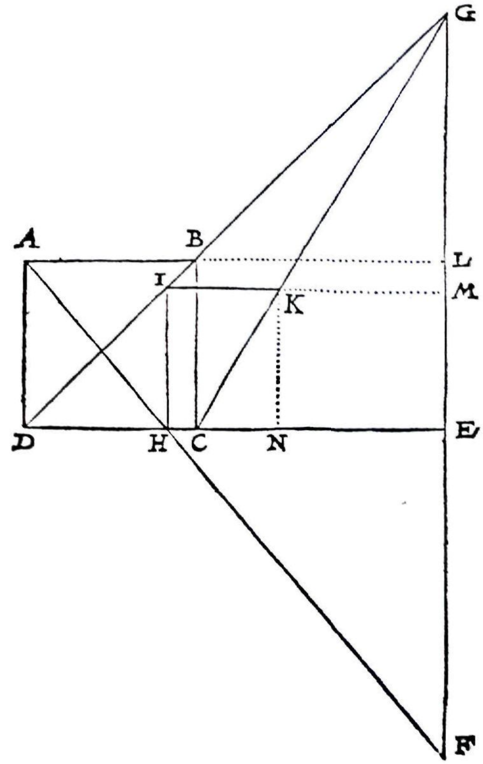


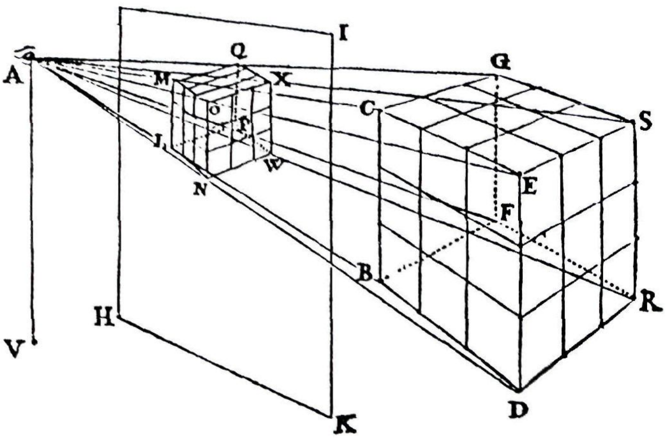
Illustration 11 – It is given that ABCD is a square with side length 2 feet and that DC lies on the ground line. The orthogonal projection of the eye upon the ground plane, F, is given by the lengths CE = 3 feet and EF = 4 feet; moreover, the distance between the eye and F is given to be 5 feet. Required are the angles and sides in the perspective image DCIK of the square. Stevin found that angle IDC = 45^0, angle DCK = $120^0 58'$, angle CKI = $59^0 2'$, angle KID = 135^0, CK = $\surd 34/9$, KI = $1^1/_3$, ID = $\surd 50/9$. Stevin 1605, pp. 90-91.

TRACTAET
der
PERSPECTIVE,
ofte
SCHYNBAERE TEYCKEN-KONST.

Waer in de Fondamenten derſelve Konſt op het kortſte verhandelt en be-toont worden.

Beſchreven door

FRANCISCUS van SCHOOTEN,
Profeſſor Matheſeos in de Univerſiteyt tot *Leyden*.



t'AMSTERDAM,

By GERRIT van GOEDESBERGH,
Boeck-verkooper op 't Water / in de Delffsche Bybel / tegen over de Nieuwe-Brugh. ANNO, 1660.

Illustration 12 – Title page of van Schooten's treatise on perspective.

Van Schooten		Stevin	Van Schooten's theorem concerns
Theorem I	contains	Theorem I	the image of a line
Theorem II		Theorem III, part 1	the images of a set of parallel line segments that are not parallel to the glass
Theorem III		Theorem III, part 2	as above, but for horizontal line segments
Theorem IV	contained in	Theorem II	line segments parallel to the ground line
Theorem V	contained in	Theorem II	vertical line segments.

Illustration 13 – A comparison of Stevin's and van Schooten's theorems.

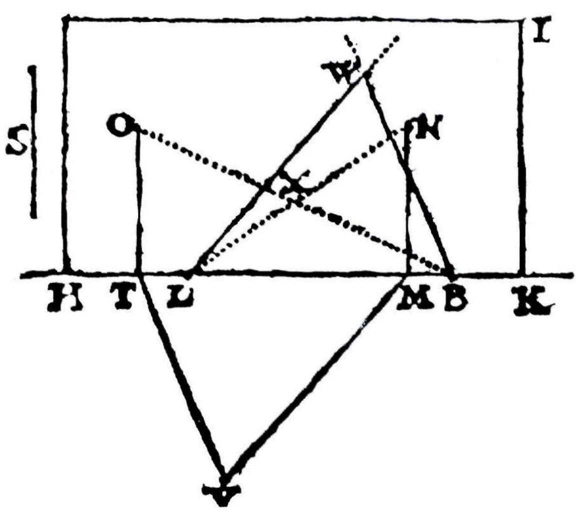


Illustration 14 – Van Schooten's construction of the perspective image of a point W lying in the ground plane. HK is the ground line, V is the orthogonal projection of the eye upon the ground plane and the line segment S indicates the distance from V to the eye. Two lines VT and VM not passing through W are drawn, they meet HK in T and M. Parallel to VT and VM are drawn WB and WL meeting HK in B and L. On the perpendiculars to HK the line segments TO and MN are made equal to S (implying that O and N are the vanishing points of WB and WL respectively). The lines OB and NL are drawn, their meeting point X is the requested image of W; van Schooten 1660, p. 529.

Daer is gegeven , een teyckentlick punt in de vloer als A , ende de glasgrondt VT, waer op het glas rechthoeckig wort verdacht, als meede de ſienders lengte ofte hooghte gelijck S H rechthoeckig op S, nu begeert men A ſijn afteyckening te vinden.

Gegeven ſijnde een teyckenlick punt in de vloer , de glas-grondt, waer op het glas recht-houckig verdacht wort op de vloer, de voet, en ſienders lengte : ſijn afteyckening te vinden.

Weſende ghegeven een verſchaeulick punt inde vloer, t'glas rechthouckich op de vloer, de voet, en de ſiender-lijn: Sijn ſchaeu te vinden.

Illustration 15 – Mohr's, van Schooten's, and Stevin's Dutch formulations of the following problem. Let a point in the ground plane, the ground line, the orthogonal projection of the eye point onto the ground plane and the distance between the two latter points be given, determine the image of the given point in a glass that is perpendicular to the ground plane; Mohr 1672, p. 32; van Schooten 1660, p. 529; Stevin 1605, p. 21.

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